CHAPTER 5

Power System Stability

5.1 Introduction

The present day electric power system network forms a complete, nonlinear dynamical system. Several controls are provided in order to perform the power system into a proper way. During the normal operation, all these controls try to bring the system to an operating equilibrium ensuring the balance of real and reactive powers in the system. Following a disturbance, the balance of real and reactive powers gets disturbed. The dynamical power system network undergoes a transition period and may settle down to an operating equilibrium with the help of the above controls, which may or may not be the same as the predisturbance equilibrium point. The capability of the system to achieve an operating equilibrium, after disturbances, depends on its inherent strength, nature and amount of disturbances. The system becomes unstable if it is not capable of regaining the operating equilibrium. Thus, a general definition of stability is given as follows:

"The stability of a dynamical system is its property or ability to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance".

During the early part of the 20th century, the concern of the power system engineers was to maximize the real power transfer from the remotely located generating stations to the load centre. The problem of maintaining synchronous operation started when two or more generators were connected in the network to share the system power demand. The difficulty in maintaining the synchronous operations was experienced specifically in the case of severe disturbances such as network faults and outage of large generating plants. This was called the **transient stability** problem. Several practical measures were suggested to improve the transient stability including the fast exciters and protection system. Although these measures helped in improving the synchronizing capability and the transient stability limit, a few of these resulted into deterioration in system damping. With poor damping, the system becomes oscillatory unstable, called

small signal stability problem. Stabilizing controls such as power system stabilizers have been used to improve the system small signal stability, facilitating the transmission system to be utilized close to their maximum power transfer capability. The stressed operation of the power system, due to increased real power transfer capability, makes the transfer of reactive power difficult, giving rise to a new stability phenomenon known as **voltage stability**.

5.2 Classification of Power System Stability

Although the power system stability is one single phenomenon, it has been classified into various types for the ease of analysis and identifying the factors affecting the stability and hence planning the control actions for its enhancement. The classification of power system stability is shown in Figure 5.1. It is broadly classified into two types: 1. angular stability and 2. voltage stability.

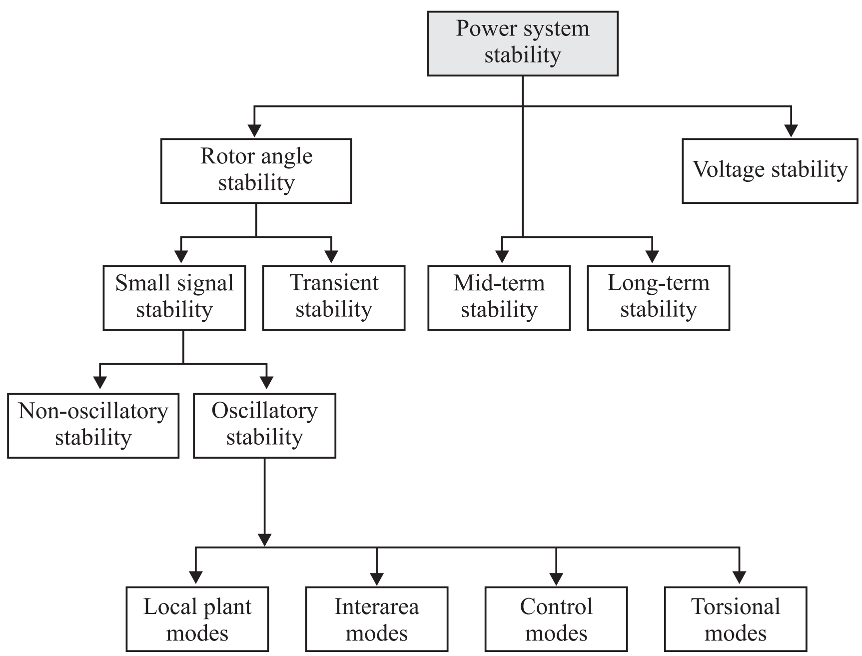


Figure 5.1 Classification of power system.

The (voltage) angle stability is the property of the interconnected power system network to maintain synchronous operation of various generating plants subjected to a disturbance. It is mainly concerned with maintaining real power requirement in the system. The stability problem involves the study of electromechanical oscillations inherent in the power systems. Disturbances in the system can be large or small, gradual or sudden. Depending on the nature of disturbance, the angle stability can be further classified as transient stability or small signal stability.

Transient stability

Transient stability refers to the large disturbances in the system. It can be defined as the **"ability of the system and its generating units to remain in synchronism following a large (severe) and sudden disturbance"**. Faults in the transmission system, sudden change of bulk load, loss of operating units, line switching are the examples of large disturbances. In general, the postdisturbance operating equilibrium is different from the predisturbance equilibrium point in case of such disturbances. Since the severe disturbances involve a large deviation in rotor angles, nonlinear dynamical model of the system is considered for the transient stability studies.

Small signal stability

Small signal stability refers to the **"ability of the system to maintain synchronism under small and sudden disturbances"**. Such disturbances occur continuously in the system due to small variation in loads and generations. Small signal instability can be either due to insufficient synchronizing torque resulting into monotonous increase in the rotor angles or due to insufficient damping torque resulting into undamped angle oscillations in the system. The stability depends on several factors such as initial operating point, transmission system strength, generator excitation and other controls in the system. Since the disturbances are small, linearized dynamical model of the system can be used for analysis. The small signal stability can be further divided into the following types.

(i) Local modes or machine-system mode, due to swinging of a generating plant unit with the rest of the system. The frequency of oscillation may range from 0.7 to 2 Hz.

(ii) Interarea mode, due to swinging of a group of coherent generating units with other group(s) of coherent units. In general, these groups are interconnected with weak tie lines. The frequency of oscillation may range from 0.1 to 0.7 Hz.

(iii) Control mode, due to the poorly tuned controllers in the system such as exciters, speed governors, HVDC converters, SVC, etc.

(iv) Torsional mode, due to interaction of mechanical oscillation of turbine generator shaft system with the oscillations in the electrical circuit, involving various controls and series compensated lines.

Voltage stability

The voltage stability also called load stability refers to the **"ability of the system to maintain load bus voltages within acceptable limit, following some disturbance or change in power demand"**.

The voltage stability can be further classified as follows.

(i) Large disturbance voltage stability, which is the ability of the system to regain voltages at all the buses within the acceptable steady state levels, when subjected to a large disturbance. The study ranges from a few seconds to tens of minutes and requires nonlinear dynamic simulation of load characteristics, and various controls including load tap changer dynamics and generator current limiters.

(ii) Small disturbance voltage stability considers small disturbances such as incremental change in loads. Static analysis is utilized to identify voltage instability conditions, various contributing factors and the stability margin.

An alternate classification of power system transient stability study is based on the time frame considered and the time response of the concerned phenomenon. It is broadly classified into three types.

(i) Short-term for a time period between 0 and 10 seconds
(ii) Mid-term for 10 seconds to a few minutes
(iii) Long-term for a few minutes to tens of minutes.

The modelling details of various power system components depend on the time frame of the phenomenon being studied. For example, the network transient is ignored in mid-term and long-term stability studies. Slow dynamics such as boiler, and on-load tap changer dynamics is required to be considered in the long-term stability study.

5.3 Transient Stability

5.3.1 Introduction

The power system transient stability has been defined as the ability of the synchronous generators in an interconnected network to remain in synchronism after being subjected to a large or severe disturbance. During a steady state operation, the generators run at a constant synchronous speed, with rotor acceleration being zero, maintaining a balance between the mechanical input power (P_m) from the turbine and the electrical power (P_e) output to the electrical network. A disturbance in the electrical network causes the (real power) output of generators to change creating an imbalance with the mechanical power input. Since the mechanical power input from turbine cannot change instantaneously, the change in electrical power requirement is met initially from the stored kinetic energy of the rotors causing the rotors to accelerate or decelerate and also changing the rotor angle position. Severe disturbances may cause a large excursion in the rotor angles. The transient stability requires

(i) the calculation of the electrical power output of generators during prefault, fault and postfault conditions. This involves solution of network power flow equations, which are nonlinear algebraic in nature.

(ii) solving the rotor dynamic equations to study the variation of rotor angle with time. The generator mechanical dynamics is described by the nonlinear differential equation known as swing equation. This requires some numerical integration method for solution.

The system is said to be stable during the postdisturbance period, if the rotors of all the machines achieve constant synchronous speed. To simplify the transient stability analysis, the classical approach has been utilized which is also discussed in this chapter.

5.3.2 Assumptions of Transient Stability Analysis

The classical technique involves the following main assumptions.

1. Mechanical input to the generator remains constant (i.e. speed governing system action is neglected).
2. Machine damping and AVR action are neglected. Synchronous machine is modelled as constant voltage source behind the transient reactance.
3. Network transients are neglected. Thus static model of network can be used.
4. Loads are represented as constant impedance/admittance type.
5. Mechanical angle of each machine rotor coincides with the electrical phase of voltage behind transient reactance.

In addition, the transmission line resistances and saliency of the synchronous machine can also be neglected, which gives conservative results. The static network equations and the machine dynamics can be formulated as given below.

5.4 Power Angle Equation of a Two Machine System

Consider a very simple system as shown in Figure 5.2.

Figure 5.2 Two machine system.

It consists of a synchronous generator supplying power to a synchronous motor through a transmission line.

To draw the reactance diagram, we know that any synchronous machine is represented by a constant voltage source in series with a reactance X. Depending upon the condition under study, the reactance may be subtransient reactance X_d'', transient reactance X_d' or steady state synchronous reactance X_d. Thus the above one line diagram can be drawn in terms of reactance diagram as shown in Figure 5.3.

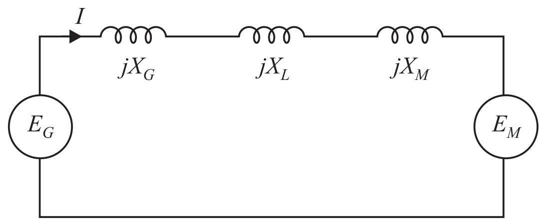


Figure 5.3 Reactance diagram.

In Figure 5.3, the generator is represented by E_G in series with X_G,

the motor is represented by E_M in series with X_M and
the transmission line with leakage reactance is given by X_L.

The total reactance between the machines is given by

$$X = X_G + X_L + X_M \tag{5.1}$$

The internal voltages E_G and E_M are generated by the flux produced by the field windings of the machines, hence their phase difference is the same as the electrical angle between the machine rotors.

The vector diagram is shown in Figure 5.4.

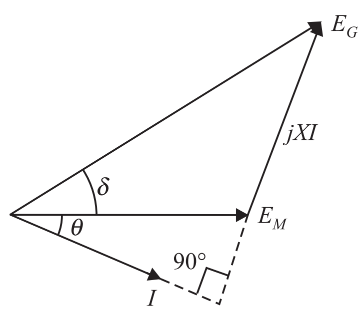


Figure 5.4 Vector diagram.

From the vector diagram,

$$\overline{E_G} = \overline{E_M} + jX\,\bar{I} \tag{5.2}$$

The equation for current is

$$\bar{I} = \frac{\overline{E_G} - \overline{E_M}}{jX} \tag{5.3}$$

Since the resistances of the machines and the transmission lines are neglected, the power output of the generator is also the power input to the motor and is given by

$$P = \text{Real part of } (\overline{E_G}^* \times \bar{I})$$

$$= \text{Re}\left[\overline{E_G}^*\left(\frac{\overline{E_G} - \overline{E_M}}{jX}\right)\right] \tag{5.4}$$

Let $\overline{E_M} = EM\angle 0$, $\quad \overline{E_G} = E_G\angle\delta$,

$\therefore \quad \overline{E_G}^* = E_G\angle{-\delta}$,

Upon substitution,

$$P = \text{Re}\left[E_G\angle{-\delta}\left(\frac{E_G\angle\delta - E_M\angle 0}{X\angle 90°}\right)\right]$$

$$= \text{Re}\left[\frac{E_G^2}{X}\angle{-90°} - \frac{E_G E_M}{X}\angle{-90°} - \delta\right] \tag{5.5}$$

$\therefore$ Real power P is given by

$$P = -\frac{E_G E_M}{X}\cos(-90° - \delta)$$

$$= -\frac{E_G E_M}{X}\cos(90° + \delta)$$

$$P = \frac{E_G E_M}{X}\sin\delta \tag{5.6}$$

The above equation shows that the power transmitted from the generator to the motor varies with the sine of the displacement angle δ between the two rotors. Hence this equation is called the power angle equation and if the curve δ against P is plotted, it is called the power angle curve as shown in Figure 5.5.

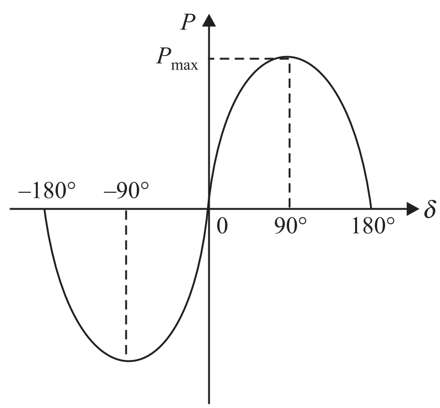


Figure 5.5 Power angle curve.

The maximum power occurs when $\sin\delta = 1$, i.e. $\delta = 90°$.

$$\therefore \quad P_{max} = \frac{E_G E_M}{X} \text{ is the steady state stability limit.}$$

When the slope $dP/d\delta$ is positive ($-90° \le \delta \le 90°$), it means that an increase in displacement angle results in an increase in transmitted power and hence the system will be stable. If $dP/d\delta$ is negative, it indicates that the system is unstable.

EXAMPLE 5.1 Two synchronous machines of equal rating have internal voltages of 1.1 + *j*0.5 and 0.8 – *j*0.4 per unit voltages respectively. The machines are connected by a line of 50 km length having only reactance and the second machine receives power of 0.9 per unit. Determine the reactance of the line per km length. Assume that there is no internal reactance for simplification.

Solution:

Given

$$\overline{E_G} = 1.1 + j0.5 = 1.21\angle 24.4°$$

$$\overline{E_M} = 0.8 - j0.4 = 0.89\angle -26.6°$$

$$P = 0.9, \text{ length of transmission line} = 50 \text{ km}$$

$$X_G = X_M = 0$$

$$\delta = 24.4° - (-26.6°) = 51°$$

We know that the power angle equation is

$$P = \frac{E_G E_M}{X} \sin \delta$$

$$0.9 = \frac{1.21 \times 0.89}{X} \sin 51°$$

$$X = \frac{0.8087}{0.9} = 0.8986 \text{ p.u.}$$

Since $X_G = X_M = 0$, X denotes the reactance of the transmission line.

$$X = 0.8986 \text{ p.u}$$

$$X \text{ per km} = \frac{0.8986}{50} = 0.0186 \text{ p.u.}$$

5.5 Power Angle Equation of a Salient Pole Machine

Consider a salient pole machine represented by means of a one line diagram as shown in Figure 5.6.

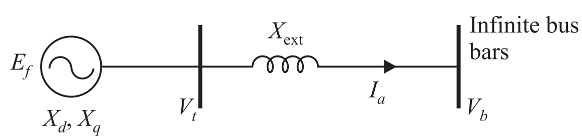


Figure 5.6 Representation of a salient pole machine.

Let E_f be the excitation emf per phase

V_t be the terminal voltage per phase

V_b be the voltage of the infinite bus

X_d be the direct axis synchronous reactance

X_q be the quadrature axis synchronous reactance

X_{ext} be the reactance between generator and infinite bus

The electrical power output of a salient pole generator is given by,

$$P_e = \underset{\text{(Excitation power)}}{\frac{E_f V_b}{X_d} \sin\delta} + \underset{\text{(Reluctance power)}}{\frac{V_b^2 (X_d - X_q)}{2 \; X_d X_q} \sin 2\delta} \tag{5.7}$$

The excitation power is the same as the power angle equation of a simple two machine system. The reluctance power varies as $\sin 2\delta$ with the maximum value of $\delta = 45°$. This term is independent of field excitation and would be present even if the field is unexcited. The reluctance component is of the order of 10 to 20 per cent of the excitation component. The reluctance component of power is usually neglected in the steady state stability studies.

For a non-salient pole machine, $X_d = X_q$, we get the original power angle equation of a simple two machine system.

5.6 Swing Equation

The behaviour of a synchronous machine during the transient period is described by the swing equation.

We know that the torque exerted on a rotating body is given by the product of moment of inertia J (kg·m^2) and angular acceleration α (rad/s^2), that is,

$$T_a = J\alpha = J \frac{d^2\theta}{dt^2} \tag{5.8}$$

where θ is the angular position of the rotor in radians at any instant of time, and t is the time in seconds.

It is convenient to measure θ with respect to a reference axis that is rotating at the synchronous speed. If δ is the angular displacement of the rotor in electrical degrees from the synchronously rotating reference axis and ω_s is the synchronous speed in electrical radians, then θ can be expressed as the sum of: (1) time varying angle $\omega_s t$ on the rotating reference axis, and (2) the torque angle δ of the rotor with respect to the rotating reference axis. In other words,

$$\theta = \omega_s t + \delta \qquad \text{electrical radians} \tag{5.9}$$

Differentiating with respect to t, we get

$$\frac{d\theta}{dt} = \omega_s + \frac{d\delta}{dt} \tag{5.10}$$

Differentiating once again with respect to t, we get

$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \tag{5.11}$$

$\therefore$ Angular acceleration of rotor, $\alpha = \dfrac{d^2\theta}{dt^2}$

$$\alpha = \frac{d^2\delta}{dt^2} \quad \text{electrical radians} \tag{5.12}$$

In a synchronous generator, the accelerating torque T_a is equal to the difference of input shaft torque T_m and the output electromagnetic torque T_e.

$$\therefore \qquad T_a = T_m - T_e$$

$$J \cdot \frac{d^2\theta}{dt^2} = J \cdot \frac{d^2\delta}{dt^2} \tag{5.13}$$

$$\therefore \qquad T_a = T_m - T_e = J \cdot \frac{d^2\delta}{dt^2}$$

Multiplying both sides by angular velocity, ω, we get

$$\omega T_a = \omega T_m - \omega T_e = J\omega \frac{d^2\delta}{dt^2} \tag{5.14}$$

If P_a, P_m and P_e denote the accelerating power, mechanical power input and electrical power output respectively, we get

$$P_a = P_m - P_e = J\omega \frac{d^2\delta}{dt^2} \tag{5.15}$$

Also, since angular momentum $M = J\omega$, therefore Eq. (5.15) can be written as

$$P_a = P_m - P_e = M \frac{d^2\delta}{dt^2} \tag{5.16}$$

Equation (5.16) is called the swing equation. It is a nonlinear differential equation of the second order.

5.6.1 Swing Curves

The solution of swing equation gives the variation of δ (in electrical radians) with respect to time (in seconds). The graph when plotted is called a swing curve as shown in Figure 5.7. It provides information regarding stability. They show the tendency of δ to oscillate and/or increase beyond the point of return. If δ increases continuously with time, the system is unstable. While if δ starts decreasing after reaching a maximum value, it is inferred that the system will remain stable.

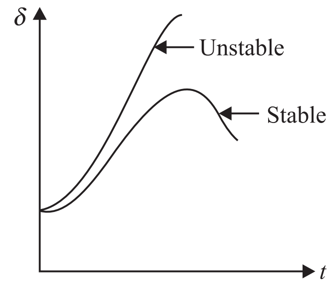


Figure 5.7 Swing curves.

5.6.2 Constants Used in Stability Analysis

Inertia constant, M

From the swing equation $M \dfrac{d^2\delta}{dt^2} = P_a$

If power is in W, δ is in rad and t is in s, then M is in W/rad/s^2 or W·s^2/rad. Since 1 J = 1 W·s, M can have its unit as J·s/rad. If power is in MW, then M is MJ·s/rad.

If δ is specified in electrical degrees, then M has unit as MJ·s/electrical degree.

If power is in pu, then M has pu power s^2/electrical degree. The value of M will be referred to as per unit value of M.

In general, constant M may be defined as the power in MW required to produce unit angular acceleration.

Kinetic energy, N

The kinetic energy of the rotor at synchronous speed denoted by N is given by

$$N = \frac{1}{2} M\omega_s$$

where $\omega_s = 2\pi\, n_s$; mechanical rad/s
$= 2\pi f$; electrical rad/s
$= 360\, f$; electrical degree/s

where n_s is the speed in rps and f is the frequency in Hz.

Normally generators of the same MVA ratings may have different values of kinetic energy and momentum. To express them in a common way, we use a constant H (also called inertia constant).

Inertia constant, H

It is defined as the ratio of stored kinetic energy to volt ampere rating of machine.

$$H = \frac{\text{kinetic energy}}{\text{MVA rating}} \quad \text{MJ/MVA}$$

$$= \frac{N}{S} \quad \text{where } S \text{ is the MVA rating}$$

$$\therefore \qquad N = SH$$

Equivalent H constant

Consider a system in which 'n' number of generators are connected in parallel to the same bus bar.

Let $S_1, S_2, S_3, \ldots, S_n$ be the MVA rating of individual machines
$H_1, H_2, H_3, \ldots, H_n$ be the inertia constants of individual machines
$N_1, N_2, N_3, \ldots, N_n$ be the kinetic energy stored in individual machines
S_e be the MVA rating of equivalent machine
H_e be the inertia constant of equivalent machine
N_e be the kinetic energy stored in equivalent machine and S_b be the base MVA.

The energy stored by the equivalent machine is given by the sum of energies stored by individual machines.

$$N = N_1 + N_2 + \cdots + N_n$$

$$S_e H_e = S_1 H_1 + S_2 H_2 + \cdots + S_n H_n$$

where

$$S_e = S_1 + S_2 + \cdots + S_n$$

If the base MVA, S_b, is equal to the combined MVA rating of individual machines S_e, i.e. $S_b = S_e$, we get

$$H_e = H_1\left(\frac{S_1}{S_b}\right) + H_2\left(\frac{S_2}{S_b}\right) + \cdots + H_n\left(\frac{S_n}{S_b}\right)$$

If the machines are identical, we have

$$S_1 = S_2 = \cdots = S_n = S$$

$$H_1 = H_2 = \cdots = H_n = H$$

then

$$H_e = n \times \left(\frac{HS}{nS}\right)$$

With identical machines

$$S_b = S_e = n \times S$$

Substituting S_b, we get

$$H_e = n \times \left(\frac{HS}{nS}\right)$$

$$H_e = H$$

Thus the equivalent H constant of several identical machines operating in parallel is the same as that of any one of the machines.

Equivalent M constant of two machines

Two synchronous machines connected by a reactance can be replaced by one equivalent machine connected through a reactance to an infinite bus as follows. The swing equation of machine 1 is given by

$$M_1 \frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \tag{5.17}$$

The swing equation of machine 2 is given by

$$M_2 \frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \tag{5.18}$$

From Eq. (5.17)

$$\frac{d^2\delta_1}{dt^2} = \frac{P_{m1} - P_{e1}}{M_1} \tag{5.19}$$

From Eq. (5.18)

$$\frac{d^2\delta_2}{dt^2} = \frac{P_{m2} - P_{e2}}{M_2} \tag{5.20}$$

Subtracting Eq. (5.20) from Eq. (5.19), we get

$$\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} = \frac{P_{m1} - P_{e1}}{M_1} - \frac{P_{m2} - P_{e2}}{M_2}$$

We can write

$$\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \frac{M_2(P_{m1} - P_{e1}) - M_1(P_{m2} - P_{e2})}{M_1 M_2} \tag{5.21}$$

If δ is the relative angle between the rotors of the two machines, then $\delta = \delta_1 - \delta_2$. We can write Eq. (5.21) as

$$\frac{d^2\delta}{dt^2} = \frac{M_2 P_{m1} - M_1 P_{m2}}{M_1 M_2} - \frac{M_2 P_{e1} - M_1 P_{e2}}{M_1 M_2} \tag{5.22}$$

Multiplying both sides of the above equation by $M_1 M_2/(M_1 + M_2)$, we get

$$\frac{M_1 M_2}{M_1 + M_2}\frac{d^2\delta}{dt^2} = \frac{M_1 M_2}{M_1 + M_2}\left(\frac{M_2 P_{m1} - M_1 P_{m2}}{M_1 M_2} - \frac{M_2 P_{e1} - M_1 P_{e2}}{M_1 M_2}\right)$$

$$\frac{M_1 M_2}{M_1 + M_2}\frac{d^2\delta}{dt^2} = \frac{M_2 P_{m1} - M_1 P_{m2}}{M_1 + M_2} - \frac{M_2 P_{e1} - M_1 P_{e2}}{M_1 + M_2} \tag{5.23}$$

The swing equation of an equivalent machine is given by

$$M' \frac{d^2\delta}{dt^2} = P'_m - P'_e \tag{5.24}$$

From Eq. (5.24) we can say that the equivalent values of M', P'_m and P'_e are given by

$$M' = \frac{M_1 M_2}{M_1 + M_2}$$

$$P'_m = \frac{M_2 P_{m1} - M_1 P_{m2}}{M_1 + M_2}$$

and

$$P'_e = \frac{M_2 P_{e1} - M_1 P_{e2}}{M_1 + M_2}$$

Relationship between inertia constant M and inertia constant H

We have

$$M = \frac{2N}{\omega_s} = \frac{2N}{360f} = \frac{N}{180f}$$

$$= \frac{SH}{180f} \text{ MJ}\cdot\text{s/electrical degree} \qquad (\text{since } N = SH)$$

If the angle is expressed in radians

$$M = \frac{SH}{\pi f} \text{ MJ}\cdot\text{s/electrical radian}$$

EXAMPLE 5.2 The moment of inertia of a 4 pole, 100 MVA, 11 kV, 3-ϕ, 0.8 power factor, 50 Hz turbo alternator is 10000 kg·m^2. Calculate H and M.

Solution:

$$J = 10000 \text{ kg}\cdot\text{m}^2$$

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$n_s = \frac{N_s}{60} = \frac{1500}{60} = 25 \text{ rps}$$

$$\omega_s = 2\pi n_s = 50\pi$$

$$N = \frac{1}{2} J\omega_s^2 = \frac{1}{2} \times 10000 \times (50\pi)^2 = 123.37 \text{ MJ}$$

$$H = \frac{N}{S} = \frac{123.37}{100} = 1.2337 \text{ MJ/MVA}$$

$$M = \frac{SH}{180f} = \frac{100 \times 1.2337}{180 \times 50} = 0.0137 \text{ MJ}\cdot\text{s/electrical degree}$$

EXAMPLE 5.3 A 50 Hz, 4 pole, turbo alternator rated 100 MVA, 11 kV has an inertia constant of 8 MJ/MVA. Determine

1. the energy stored in the rotor at synchronous speed.
2. find the rotor acceleration if the mechanical input is suddenly raised to 80 MW for an electric load of 50 MW. (Neglect mechanical and electrical losses).

Solution:

(i) H = 8 MJ/MVA; S = 100 MVA

We know that

$$N = HS = 800 \text{ MJ}$$

(ii) Swing equation is $M\dfrac{d^2\theta}{dt^2} = P_a = P_m - P_e$

Here for alternator, P_m = 80 MW

P_e = 50 MW

$\therefore$ P_a = 30 MW

Also $M = \dfrac{SH}{180f} = \dfrac{N}{180f} = \dfrac{800}{180 \times 50} = 0.0889 \text{ MJ}\cdot\text{s}$

$\therefore$ Acceleration= $\dfrac{d^2\delta}{dt^2} = \dfrac{P_a}{M} = \dfrac{30}{0.0889} = 337.5$ electrical degree/s

EXAMPLE 5.4 A 50 Hz, 4 pole turbo generator rated 20 MVA, 11 kV has an inertia constant of H = 9 kW·s/kVA. Find the kinetic energy stored in the rotor at synchronous speed. Find the acceleration, if the input less the rotational losses is 26800 HP and the electrical power developed is 16 MW.

Solution:

S = 20 MVA, 11 kV,

$$H = 9 \text{ kW}\cdot\text{s/kVA} = 9 \text{ kJ/kVA}$$

$$H = 9 \text{ MJ/MVA}$$

Kinetic energy = $N = HS = 9 \times 20 = 180$ MJ

$$P_a = P_m - P_e$$

$$P_m = 26800 \times 746 = 19992800 \text{ W} = 19.99 \text{ MW}$$

$$P_a = 19.99 - 16 = 3.99 \text{ MW}$$

$$M = \frac{N}{180f} = \frac{180}{180 \times 50} = 0.02 \text{ MW}\cdot\text{s}^2/\text{electrical degree}$$

We know that

$$M\frac{d^2\delta}{dt^2} = P_a \quad \Rightarrow \quad \frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{3.99}{0.02}$$

$$\text{Acceleration } \frac{d^2\delta}{dt^2} = 199.5 \text{ electrical degree/s}^2$$

EXAMPLE 5.5 A power station with 4 generators each 80 MVA, 8 MJ/MVA is in proximity with another power station having 3 generators each 200 MVA, 3.5 MJ/MVA. Determine the inertia constant of a single equivalent machine for use in stability studies. Assume a base value of 100 MVA.

Solution:

4 generators are each of 80 MVA, 8 MJ/MVA

3 generators are each of 200 MVA, 3.5 MJ/MVA

We know that

$$H_eS_e = H_1S_1 + H_2S_2 + \cdots$$

$$= \sum_{i=1}^{4} H_1S_1 + \sum_{i=1}^{3} H_2S_2$$

$$= (4 \times 80 \times 8) + (3 \times 200 \times 3.5)$$

$$= 2560 + 2100 = 4660 \text{ MJ}$$

$$H_e = \frac{4660}{S_e} = \frac{4660}{100} = 46.6 \text{ MJ/MVA}$$

EXAMPLE 5.6 Two turbo alternators specified below are interconnected using a short line:

Machine 1 : 4 poles, 50 Hz, 125 MVA, 0.8 lag, 25000 kg·m^2

Machine 2 : 4 poles, 50 Hz, 150 MVA, 0.9 lag, 20000 kg·m^2

Determine the inertia constant of the single equivalent machine on a base of 150 MVA.

Solution:

Machine 1

$$M_1 = \frac{N_1}{180f}$$

$$N_1 = \frac{1}{2}J\omega_s^2$$

$$N_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$n_s = \frac{1500}{60} = 25 \text{ rps}$$

$\therefore$
$$N_1 = \frac{1}{2} \times 25000 \times \omega_s^2$$

$$\omega_s = 2\pi\, n_s = 157.0796 \text{ rad/s} = 308.425 \text{ MJ}$$

$$M_1 = \frac{N_1}{180f} = 0.03426 \text{ MJ}\cdot\text{s/electrical degree}$$

$$M_1 \text{ in p.u. on a base of 150 MVA} = 2.2846 \times 10^{-4} \text{ p.u.}$$

Machine 2

$$n_s = 25 \text{ rps}$$

$$N_2 = \frac{1}{2}J\omega_s^2 = \frac{1}{2} \times 20000 \times (157.0796)^2 = 246.74$$

$$M_2 = \frac{N_2}{180f} = 0.274155 \text{ MJ}\cdot\text{s/electrical degree}$$

M_2 in p.u on a base of 150 MVA = 1.8277×10^{-4} p.u.

$$M = \frac{M_1 M_2}{M_1 + M_2} = 1.01538 \times 10^{-4} \text{ p.u.}$$

EXAMPLE 5.7 A generator *A* is rated at 50 Hz, 60 MW, 75 MVA, 1500 rpm and has an inertia constant H = 7 MJ/MVA. The corresponding data for another generator *B* is 50 Hz, 120 MW, 133.3 MVA, 3000 rpm, and 4 MJ/MVA.

(a) If these two generators operate in parallel in a power station, calculate H for the equivalent generator on a base of 100 MVA.
(b) If the power station is connected to another power station which has two of each type of generator, calculate H for the equivalent generator connected to an infinite bus bar.

Solution:

(a) We know that

$$S_e H_e = H_1 S_1 + H_2 S_2$$
$$H_e S_e = (7 \times 75) + (4 \times 133.3)$$
$$H_e = \frac{(7 \times 75) + (4 \times 133.3)}{100} = 10.582 \text{ MJ/MVA}$$

(b) Another power station has two of each type of generator. So the equivalent H_e becomes twice the original H_{e1}.

$$H_{e2} = 2 \times 10.582 = 21.164 \text{ MJ/MVA}$$

When they are connected in parallel, the equivalent machine

$$H_e = \frac{H_{e1} \cdot H_{e2}}{H_{e1} + H_{e2}} = 7.055 \text{ MJ/MVA}$$

EXAMPLE 5.8 Two turbo alternators given below are interconnected using a short line.

Machine 1 : 4 poles, 50 Hz, 75 MVA, 0.8 lag, 30000 $\text{kg}\cdot\text{m}^2$
Machine 2 : 2 poles, 50 Hz, 100 MVA, 0.85 lag, and 10000 $\text{kg}\cdot\text{m}^2$

Determine the inertia constant of the single equivalent machine on a base of 200 MVA.

Solution:

Machine 1

$$\omega_s = 2\pi\, n_s = 2\pi \times \frac{120f}{4 \times 60} = 157.079$$

$$\text{Kinetic energy} = \frac{1}{2}J\omega_s^2 = \frac{1}{2}\times 30000\times(157.079)^2$$

$$N_1 = 370$$

$$H_1 = \frac{N_1}{S_1} = 4.935 \text{ MJ/MVA}$$

Machine 2

$$\omega_s = 314.159$$

$$N_2 = \frac{1}{2}J\omega_s^2 = \frac{1}{2}\times 10000\times(314.159)^2 = 493.48$$

$$H_2 = \frac{N_2}{S_2} = \frac{493.48}{100} = 4.9348 \text{ MJ/MVA}$$

Equivalent inertia constant

$$M_1 = \frac{SH_1}{180f} = \frac{\frac{75}{200}\times 4.935}{180\times f} = 2.056\times 10^{-4} \text{ p.u.}$$

$$M_2 = \frac{SH_2}{180f} = 2.738\times 10^{-4} \text{ p.u.}$$

$$M = \frac{M_1M_2}{M_1+M_2} = 1.174\times 10^{-4} \text{ p.u.}$$

5.6.3 Determination of Change in Rotor Angle When the Machine is Loaded

The swing equation is given by

$$M\frac{d^2\delta}{dt^2} = P_a$$

$$\therefore \qquad \frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

where P_a/M is a constant.

Multiplying both sides by $2\dfrac{d\delta}{dt}$

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{P_a}{M}\cdot 2\frac{d\delta}{dt}$$

$$= \frac{2P_a}{M}\cdot\frac{d\delta}{dt}$$

$$\frac{d}{dt}\left[\frac{d\delta}{dt}\right]^2 = 2\frac{P_a}{M}\frac{d\delta}{dt}$$

Upon integration,

$$\int d\left(\frac{d\delta}{dt}\right)^2 = 2\frac{P_a}{M}\int d\delta$$

$$\left(\frac{d\delta}{dt}\right)^2 = 2\frac{P_a}{M}\cdot\delta$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2P_a}{M}}\,\delta^{1/2}$$

$$\frac{d\delta}{\delta^{1/2}} = \sqrt{\frac{2P_a}{M}}\cdot dt \qquad (5.25)$$

Integrating once again,

$$\int \delta^{-1/2}\,d\delta = \int\sqrt{\frac{2P_a}{M}}\cdot dt$$

$$\frac{\delta^{-1/2+1}}{-\frac{1}{2}+1} = \sqrt{\frac{2P_a}{M}}\cdot t$$

$$\frac{\delta^{1/2}}{\frac{1}{2}} = \sqrt{\frac{2P_a}{M}}\cdot t \qquad (5.26)$$

$$\delta^{1/2} = \frac{1}{2}\sqrt{\frac{2P_a}{M}}\cdot t = \sqrt{\frac{P_a}{2M}}\cdot t$$

Substituting Eq. (5.26) in Eq. (5.25), we get

$$\frac{d\delta}{dt} = \sqrt{\frac{2P_a}{M}}\cdot\sqrt{\frac{P_a}{2M}}\cdot t = \sqrt{\frac{2P_a^2}{2M^2}}\cdot t$$

$$\frac{d\delta}{dt} = \left(\frac{P_a}{M}\right)t$$ is the change in rotor angle at anytime

EXAMPLE 5.9 The rotor of an alternator is subjected to an acceleration of 15 electrical rad/s^2. If this acceleration exists constantly, compute the change in rotor angle and the speed in rpm at the end of 5 cycles. H = 5 MJ/MVA. Frequency = 50 Hz. Initially the machine is running at a normal speed without any acceleration. Number of poles = 2.

Solution:

Given the acceleration $\frac{d^2\delta}{dt^2}$ = 15 electrical rad/s^2

From swing equation, $\frac{d^2\delta}{dt^2} = \frac{P_a}{M}$ = 15 electrical rad/s^2

Change in rotor angle = $\dfrac{d\delta}{dt} = \dfrac{P_a}{M} \cdot t$

Time period for five cycles = no. of cycles × time period for the given frequency

$$= 5 \times \frac{1}{50} = 0.1 \text{ s}$$

∴ Change in rotor angle = $\dfrac{d\delta}{dt}$ = 15 × 0.1 = 1.5 electrical rad/s

$$\frac{d\delta}{dt} = 1.5 \text{ electrical rad/s}$$

Since pole pair p = 1, we have $\theta_m = \theta_e$

∴ $\dfrac{d\delta}{dt}$ = 1.5 mechanical rad/s = $2\pi\, n_s$

$$n_s = \frac{1.5}{2\pi} = 0.2387 \text{ rps}$$

∴ N_s = 0.2387 × 60 = 14.32 rpm

Since the machine is subjected to constant acceleration, the speed will increase by 14.32 rpm.

∴ New speed = synchronous speed + N_s

$$\text{Synchronous speed} = \frac{120f}{p} = 3000 \text{ rpm}$$

∴ New speed = 3014.32 rpm

EXAMPLE 5.10 A 200 MVA, 11 kV, 50 Hz, 4 pole turbo alternator has an inertia constant of 6 MJ/MVA.

(i) Determine the stored energy in the rotor at synchronous speed.
(ii) The machine is operating at a load of 120 MW when the load suddenly increases to 160 MW. Determine the rotor retardation. Neglect losses.
(iii) The retardation calculated above is maintained for 5 cycles. Determine the change in power angle and the rotor speed in rpm at the end of this period.

Solution:

H = 6 MJ/MVA, S = 200 MVA

(i) $N = HS$ = 6 × 200 = 1200 MJ

(ii) P_a = 40 (retardation)

$$M = \frac{N}{180f} = \frac{1200}{180 \times 50} = 0.1333 \text{ MJ} \cdot \text{s/ electrical degree}$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{40}{0.1333} = 300 \text{ electrical degree/s}^2$$

We know that change in rotor angle

$$\frac{d\delta}{dt} = \frac{P_a}{M} \cdot t$$

where t is the time period for acceleration or retardation.

(iii) Here for five cycles $t = 5 \times \frac{1}{50} = 0.1\,\text{s}$

$$\therefore \quad \frac{d\delta}{dt} = 300 \times 0.1 = 30 \text{ electrical degree/s}$$

$$= 30 \times \frac{\pi}{180} \text{ electrical rad/s} = 0.5236 \text{ electrical rad/s}$$

$$= \frac{0.5236}{p} \text{ mechanical rad/s}$$

Pole pairs, $p = 2$

$$\therefore \quad \frac{d\delta}{dt} = \frac{0.5236}{2} = 0.2618 \text{ mechancial rad/s } = 2\pi n_s$$

$$n_s = \frac{0.2618}{2\pi} = 0.04167 \text{ rps}$$

$$N_s = n_s \times 60 = 2.5 \text{ rpm}$$

Since the retardation is for 5 cycles,

New speed = synchronous speed – N_s

$\therefore$ New speed = 1500 – 2.5 = 1497.5 rpm

5.7 Equal Area Criterion

In a system to determine whether a power system is stable after a disturbance, it is necessary to plot the swing curve. If this curve shows that the angle between any two machines tends to increase without limit, the system is unstable. If after all disturbances, the angle between the two machines reaches the maximum value and thereafter decreasing, thus oscillating with constant amplitude, it is probable although not certain that the system is stable. There is a simple graphical method of determining whether the machines come to rest with respect to each other. This method is known as the equal area criterion for stability.

The principle by which stability under transient conditions is determined without solving the swing equation is called the equal area criterion of stability.

When this method is used, it completely eliminates the need of computing swing curves and hence saves a considerable amount of work. The derivation of the equal area criterion is made for one machine and an infinite bus although the method can be adapted to a two-machine system.

Although not applicable to a multi-machine system, the method helps in understanding how certain factors influence the transient stability of any system.

5.7.1 Assumptions Made in Equal Area Criterion

1. Constant input over the time interval being considered.
2. Damping effect is neglected.
3. Constant voltage behind transient reactance.

In a two-machine system, it is quite likely to be stable, if it survives the first swing.

Validity of the assumptions

1. The assumption of constancy of the input power is justified by the fact that the effect of the action of governors on the input will not be appreciable up to the first swing, and thereafter, it will only tend to restore stability.
2. The effect of damping would be to slightly reduce the amplitude of the first swing and would therefore tend to diminish the amplitude of any subsequent swing.

When a fault occurs in a system, the fault current tends to cause demagnetization of the field due to armature reaction and the field current increases to the extent required to offset this effect so as to keep up the constant flux linkage of the field circuit. If the machine is not provided with a voltage regulator, the field current decreases to its original value, as also the flux linkage. As the time constant may be a few seconds, during the first swing there will not be any appreciable decrease in the flux linkage. Even if a voltage regulator is provided, it will take some time before the regulator and the exciter response becomes effective. During the subsequent swings, however, the voltage regulator will contribute substantially toward the maintenance of stability. It is, therefore, reasonable to assume constant voltage behind transient reactance without any significant error.

5.7.2 Equal Area Criterion Method Applied to a Machine Swinging with Respect to an Infinite Bus

The swing equation of a synchronous machine swinging with respect to an infinite bus is given by

$$M\frac{d^2\delta}{dt^2} = P_a = P_m - P_e$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

Multiplying both sides by $2\frac{d\delta}{dt}$

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = 2\frac{P_a}{M}\frac{d\delta}{dt}$$

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = 2\frac{P_a}{M}\frac{d\delta}{dt}$$

$$d\left[\left(\frac{d\delta}{dt}\right)^2\right] = 2\frac{P_a}{M}d\delta$$

Upon integration,

$$\left(\frac{d\delta}{dt}\right)^2 = \int_{\delta_0}^{\delta_t} 2\frac{P_a}{M}d\delta$$

$$= \frac{2}{M}\int_{\delta_0}^{\delta_t} P_a d\delta$$

$$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta_t} P_a d\delta}$$

If $d\delta/dt = \omega$, the velocity of the displacement angle with respect to the infinite bus, then

$$\omega = \frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_0}^{\delta_t} P_a d\delta}$$

If the machine is continuously swinging, then the above equation will be non-zero. The stability is indicated by the zero value, i.e. if the integral vanishes.

i.e. $$\int P_a d\delta = 0$$

$$\int (P_m - P_e)\, d\delta = 0$$

$$\int P_m d\delta - \int P_e d\delta = 0$$

$\therefore$ $$\int P_m d\delta = \int P_e d\delta$$

Under the steady operating conditions, $P_m = P_e$ as given by the constant input line. The power angle curve is shown in Figure 5.8.

Let us assume that the initial angle be δ_0. The corresponding value of P_e is given by bf.

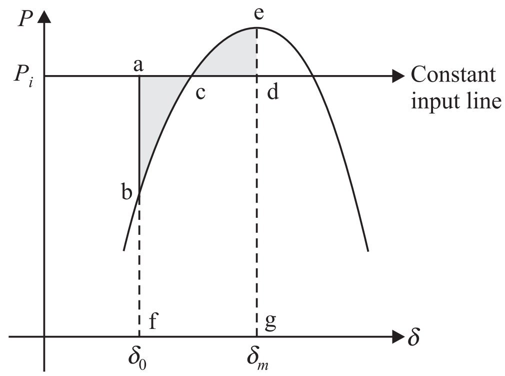


Figure 5.8 Equal area criterion.

For the stability criterion $\int P_m d\delta = \int P_e d\delta$

which is indicated by the equality of the area of the rectangle afgd and the area bcegf.

$$\text{Area afgda} = \text{Area bcedgfb}$$

$$\text{Area abc} + \text{Area bcdgf} = \text{Area ced} + \text{Area bcdgf}$$

$$\text{Area abc} = \text{Area ced}$$

$A_1 = A_2$ which is shown by the dashed line, i.e. the area A_1 below P_m line is equal to the area A_2 above P_m line. Hence this method is called equal area criterion.

From Figure 5.8, 'ab' represents $P_m - P_e \Rightarrow$ accelerating power corresponding to the initial angle δ_0, 'de' represents $P_e - P_m \Rightarrow$ decelerating power and when the accelerating power is equal to the decelerating power, the machine tends to return to the original condition if there is a balance between the two areas at $\delta = \delta_m$.

The system will remain stable only when $A_2 \geq A_1$, i.e. if accelerating area A_1 is greater than the decelerating area A_2, the system will definitely become unstable.

5.7.3 Sudden Change in Mechanical Input

Some of the conditions caused by the sudden increase in the mechanical load on a synchronous motor connected to an infinite bus can be predicted by analyzing Figure 5.9. An infinite bus is considered as a generator with infinite H constant and with constant frequency.

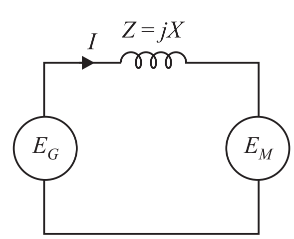


Figure 5.9 Motor connected to infinite bus.

The sinusoidal curve P_e is a plot of the electric power input to the motor with all resistance neglected. The curve P_e is plotted from the equations

$P = (|E_G||E_M|/|X|)\sin\delta$ and $P_{\max} = |E_G||E_M|/|X|$, where $|E_M|$ is the voltage of the infinite bus, $|E_G|$ is the voltage behind transient reactance of the motor, and X is determined from the transient reactance of the motor plus the reactance of the transformer and line, if any, between the motor and the infinite bus.

Originally the motor is operating at synchronous speed with a torque angle of δ_0, and the mechanical power output P_0 is equal to the electric power input P_e corresponding to δ_0. When the mechanical load is suddenly increased so that the power output is P_s, which is greater than the electric power input at δ_0, the difference in power must come from the kinetic energy stored in the rotating system. This can be accompanied only by a decrease in speed, which results in an increase in the torque angle δ. As δ increases, the electric power received from the bus increases until $P_e = P_s$ at point "b" in the curve. At this point there is equilibrium of input and output torque so that acceleration is zero, but the motor is running at less than synchronous speed so that δ is increasing. The angle δ continues to increase, but after passing through point "b" the electric power input P_e is greater than P_s, and the difference must be stored in the system through an increase in kinetic energy accompanying an increase in speed. Thus, between points "b" and "c" as δ increases, the speed also increases, until synchronous speed is again reached at point "c", where the torque angle is δ_m. At point "c", P_e is still greater than P_s and speed continues to increase, but δ starts to decrease as soon as the speed of the motor exceeds synchronous speed. The maximum value of δ is δ_m at point "c". As δ decreases, point "b" is reached again with the speed more than the synchronous speed, so that δ continues to decrease until point "b" is reached. The motor is again operating at synchronous speed, and the cycle is repeated.

When the load is suddenly increased from P_0 to P_s, the motor oscillates around the equilibrium torque angle δ_s between δ_0 and δ_m as shown in Figure 5.10. If damping is present, the oscillations decrease, and stable operation results at δ_s. Table 5.1 shows the changes in speed, angle, electric power input, mechanical power output, stored energy and acceleration or deceleration as the machine oscillates. A thorough study of Table 5.1 will lead to a better understanding of transient disturbances.

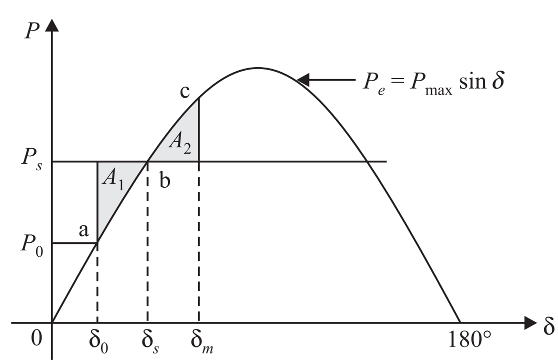


Figure 5.10 Electric power input to a motor as a function of torque angle δ.

Table 5.1 Changing conditions in a synchronous motor swinging with respect to an infinite bus because of a sudden increase in load

Position in cycle	***Motor speed ω***	***Torque angle δ***	***Electric power P_e***	***Stored energy 1/2 $J\omega_2$ = W***	***Rotating system undergoing***
At point a	$\omega = \omega_s$, decreasing	$\delta = \delta_0$, minimum	$P_e < P_s$, minimum	$\omega = \omega_s$, decreasing	Deceleration
From a towards b	$\omega = \omega_s$, decreasing	Increasing	$P_e < P_s$, increasing	$\omega < \omega_s$, decreasing	Deceleration
At point b	$\omega = \omega_s$, minimum	$\delta = \delta_s$, increasing	$P_e = P_s$, increasing	$\omega < \omega_s$, minimum	Acceleration
From b towards c	$\omega = \omega_s$, increasing	Increasing	$P_e > P_s$, increasing	$\omega < \omega_s$, decreasing	
At point c	$\omega = \omega_s$, increasing	$\delta = \delta_m$, maximum	$P_e > P_s$, maximum	$\omega = \omega_s$, increasing	Acceleration
From c towards b	$\omega = \omega_s$, maximum	Decreasing	$P_e > P_s$, decreasing	$\omega > \omega_s$, increasing	
At point b	$\omega = \omega_s$, maximum	$\delta = \delta_s$, decreasing	$P_e = P_s$, decreasing	$\omega > \omega_s$, maximum	Deceleration
From b towards a	$\omega = \omega_s$, decreasing	Decreasing	$P_e < P_s$, minimum	$\omega > \omega_s$, decreasing	

The maximum swing of the motor to a torque angle δ_m can be found by equal area criterion by equating the shaded areas A_1 and A_2.

The shaded area A_1 is given by

$$A_1 = \int_{\delta_0}^{\delta_s} (P_s - P_e)\, d\delta$$

Similarly, the shaded area A_2 is given by

$$A_2 = \int_{\delta_s}^{\delta_m} (P_e - P_s)\, d\delta$$

and

$$A_1 - A_2 = \int_{\delta_0}^{\delta_s} (P_s - P_e)\, d\delta - \int_{\delta_s}^{\delta_m} (P_e - P_s)\, d\delta$$

$$A_1 - A_2 = \int_{\delta_0}^{\delta_m} (P_s - P_e)\, d\delta$$

Equation $\int_{\delta_0}^{\delta} \frac{2(P_s - P_e)}{M} d\delta = 0$ is satisfied and $d\delta/dt = 0$ when $A_1 = A_2$. The maximum torque angle δ_m is located graphically so as to make A_2 equal to A_1.

5.7.4 Estimating Transient Stability Limit

Figure 5.11 shows a suddenly applied load, which is larger than that shown in Figure 5.12. The area A_2 above P_s under the curve P_e is less than A_1 and $d\delta/dt$ is not zero at $\delta = \delta_m$. Therefore δ continues to increase after $\delta = \delta_m$. P_e again becomes less than P_s. The torque angle δ continues to increase beyond δ_m, and restoring forces are not encountered. The system is stable only if an area A_2 located above P_s is equal to A_1. The test of equal areas is called the **equal area criterion**. The maximum allowable increase in the power suddenly taken from the motor originally supplying the power P_0 is shown in Figure 5.12. A suddenly applied load greater than that shown in Figure 5.12 would not permit the torque angle of the motor to stop increasing in magnitude before the input power becomes less than the power required, since the area above P_s would be less than A_1.

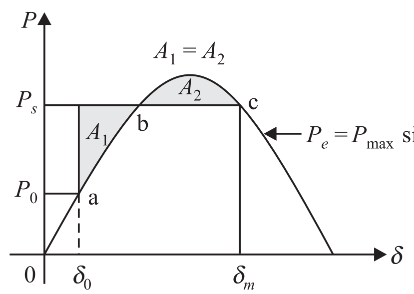


Figure 5.11 Electric power input to a motor as a function of torque angle for a suddenly increased load.

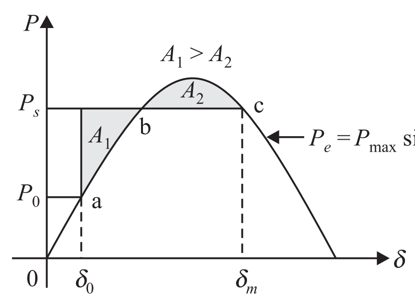


Figure 5.12 Electric power input to a motor as a function of torque angle for the maximum sudden increase of load without loss of stability.

EXAMPLE 5.11 A synchronous motor is receiving 30% of the power which is capable of receiving from an infinite bus. If the load on the motor is doubled, calculate the maximum value of δ during the swinging of the motor around its new equilibrium position.

Solution: The load in the motor is 30% and let the initial operating load angle be δ_0, i.e. $P_e = 0.3P_{max}$.

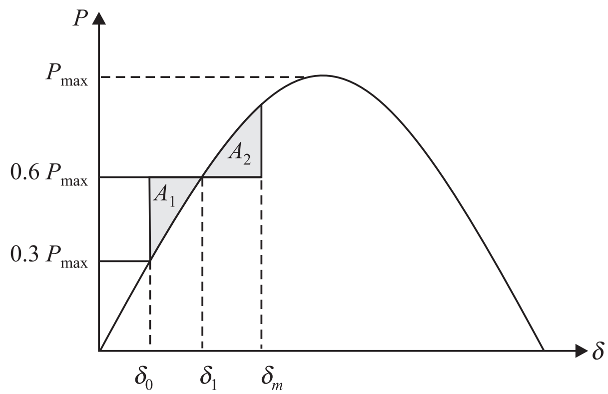


Figure 5.13

Generally,
$$P_{max} \sin \delta_0 = P_e = 0.3P_{max}$$
$$\sin \delta_0 = 0.3$$
$$\delta_0 = 17.46°$$

When the load is doubled (Figure 5.13),
$$P_e = 0.6P_{max}$$
$$\therefore \quad P_{max} \sin \delta_1 = 0.6P_{max}$$
$$\sin \delta_1 = 0.6$$
$$\delta_1 = 36.87°$$

To find the maximum value of $\delta(\delta_m)$

Using the equal area criterion, $A_1 = A_2$

$$\text{Area } A_1 = \int_{\delta_0}^{\delta_1} (0.6P_{max} - P_{max} \sin \delta)\, d\delta$$
$$= 0.6P_{max}(\delta_1 - \delta_0) + P_{max}(\cos \delta)_{\delta_0}^{\delta_1}$$
$$= 0.6P_{max}(\delta_1 - \delta_0) + P_{max}(\cos \delta_1 - \cos \delta_0)$$
$$\text{Area } A_2 = \int_{\delta_1}^{\delta_m} (P_{max} \sin \delta - 0.6P_{max})\, d\delta$$
$$= -P_{max}(\cos \delta_m - \cos \delta_1) - 0.6P_{max}(\delta_m - \delta_1)$$

Now $A_1 = A_2$

$$0.6P_{max}(\delta_1 - \delta_0) + P_{max}(\cos \delta_1 - \cos \delta_0) = -P_{max}(\cos \delta_m - \cos \delta_1) - 0.6P_{max}(\delta_m - \delta_1)$$

or $\quad 0.6(\delta_1 - \delta_0) + (\cos \delta_1 - \cos \delta_0) = \cos \delta_1 - \cos \delta_m - 0.6(\delta_m - \delta_1)$

or $\quad -0.6\delta_0 + \cos \delta_1 - \cos \delta_0 = \cos \delta_1 - \cos \delta_m - 0.6\delta_m$

or $\quad \cos \delta_m + 0.6\delta_m = 0.6\delta_0 + \cos \delta_0$

$$\cos\delta_m = \cos\delta_0 + 0.6(\delta_0 - \delta_m) \times \frac{\pi}{180}$$

$$= 0.95393 + 0.1828 - \left(\frac{0.6\,\pi\,\delta_m}{180}\right)$$

$$\cos\delta_m = 1.1368 - 0.01047\delta_m$$

Solving for δ_m by trial and error method, we get

$$\boxed{\delta_m > \delta_1}$$

$$\delta_m = 58.15°$$

5.7.5 Sudden Three Phase Fault at One End of the Transmission Line

Let us consider the three-phase short-circuit fault which occurs at the sending of the transmission line 2 of single machine connected to an infinite system with double circuit transmission line as shown in Figure 5.14.

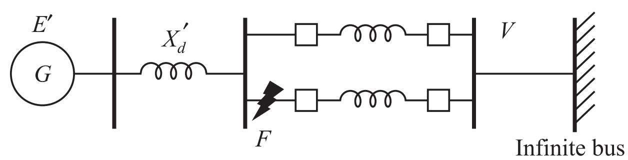


Figure 5.14 Three-phase fault at one end of the transmission line.

Prefault condition: Before the occurrence of a fault, both transmission lines are intact as shown in Figure 5.15. The power angle curve is given by

$$P_{e1} = \frac{|E'||V|}{X_I}\sin\delta = P_{\max 1}\sin\delta$$

where $X_I = X'_d + \left(\dfrac{X_{TL1} \times X_{TL2}}{X_{TL1} + X_{TL2}}\right)$

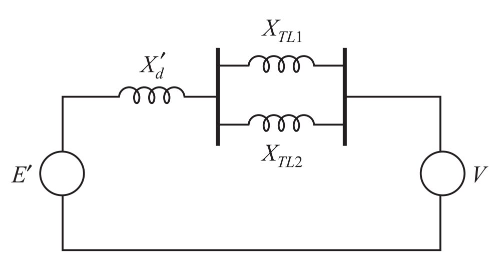


Figure 5.15 Prefault condition.

During fault condition: Upon the occurrence of three phase fault at the sending of the transmission line 2, the generator gets isolated from the power system for the purpose of power flow as shown in Figure 5.16. Thus during the period of fault lasts,

$$P_{e_2} = 0$$

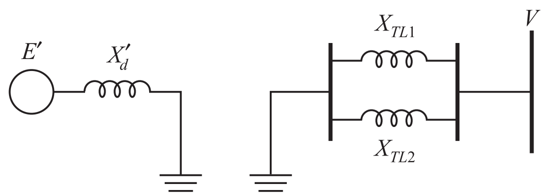


Figure 5.16 During fault condition.

The rotor therefore accelerates and δ angle increases. Synchronism will be lost unless the fault is cleared in time.

Postfault condition: The circuit breakers at the two ends of the faulted line open at time t_c (corresponding to angle δ_c), the clearing time, disconnecting the faulted line as shown in Figure 5.17. The power angle curve is given by

$$P_{e3} = \frac{|E'||V|}{X_{III}} \sin\delta = P_{\max 2} \sin\delta$$

where $X_{III} = X'_d + X_{TL1}$.

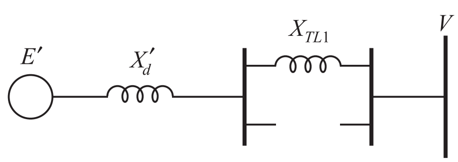


Figure 5.17 Postfault condition.

Obviously, $P_{\max 2} < P_{\max 1}$. The rotor now starts to decelerate as shown in Figure 5.18. The system will be stable if a decelerating area A_2 can be found equal to accelerating area A_1 before δ reaches the maximum allowable value $\delta_{\max}$. As area A_1 depends upon the clearing time t_c, the clearing time must be less than a certain value (critical clearing time) for the system to be stable. It is to be observed that the equal area criterion helps to determine the critical clearing angle and not the critical clearing time. Critical clearing time can be obtained by numerical solution of the swing equation.

5.7.6 Sudden Three Phase Fault at Middle of a Transmission Line

Consider the fault occurs at middle of a transmission line 2 as shown in Figure 5.19.

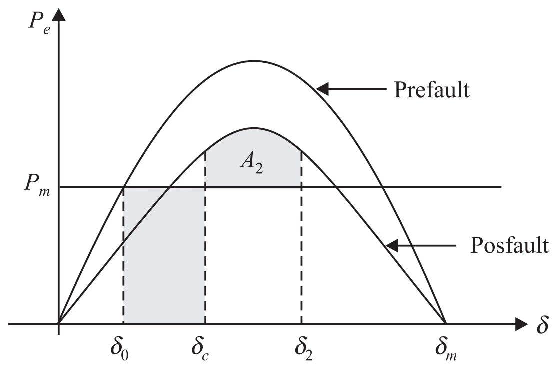


Figure 5.18

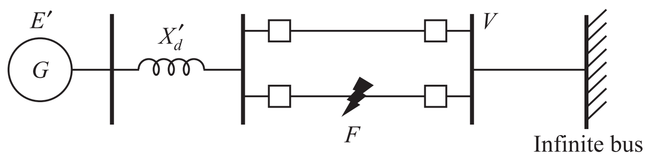


Figure 5.19 Fault at middle of transmission line.

Prefault condition: Before the occurrence of a fault, both the lines are connected as shown in Figure 5.20.

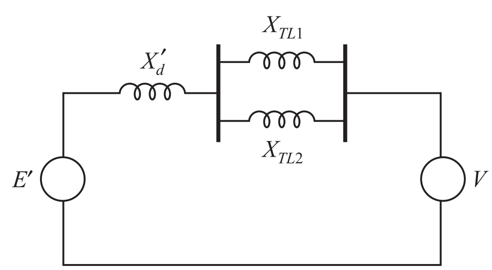


Figure 5.20 Prefault condition.

The power angle curve is given by

$$P_{e1} = \frac{|E'||V|}{X_I} \sin \delta = P_{\max 1} \sin \delta$$

where, $X_I = X'_d + \left(\frac{X_{TL1} \times X_{TL2}}{X_{TL1} + X_{TL2}} \right)$

During fault condition: The circuit model of the system during fault is shown in Figure 5.21. This circuit reduces to that of Figure 5.22 through one delta to star and one star to delta conversion.

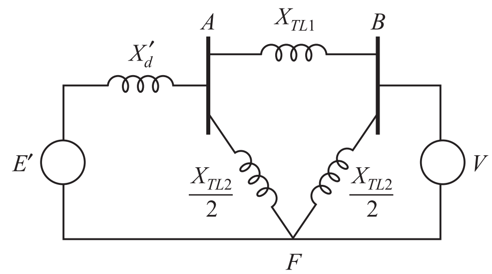


Figure 5.21 During fault.

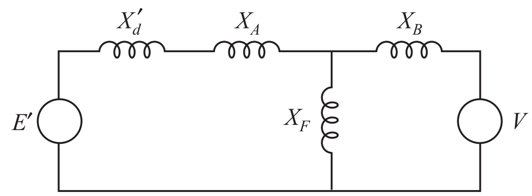


Figure 5.22 Converting reactances from delta to star.

Using delta to star conversion, the circuit becomes as shown in Figure 5.22,

$$X_A = \frac{X_{TL1} \times \dfrac{X_{TL2}}{2}}{X_{TL1} + \dfrac{X_{TL2}}{2} + \dfrac{X_{TL2}}{2}} = X_B$$

$$X_F = \frac{\dfrac{X_{TL2}}{2} \times \dfrac{X_{TL2}}{2}}{X_{TL1} + \dfrac{X_{TL2}}{2} + \dfrac{X_{TL2}}{2}}$$

Convert star connection to delta connection, the circuit becomes as shown in Figure 5.23,

$$X_{II} = \frac{(X'_d + X_A)X_F + X_F X_B + (X'_d + X_A)X_B}{X_F}$$

The power angle curve during fault is given by

$$P_{e2} = \frac{|E'||V|}{X_{II}} \sin\delta = P_{\max 2} \sin\delta$$

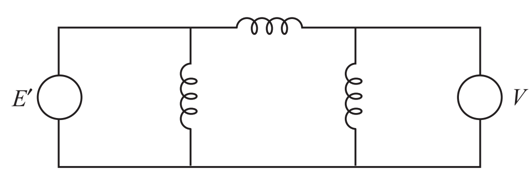


Figure 5.23 Converting reactances from star to delta.

Postfault condition: In this condition, the faulted transmission line 2 is open after fault and the circuit is shown in Figure 5.24.

$$P_{e3} = \frac{|E'||V|}{X_{III}} \sin\delta = P_{\max 3} \sin\delta$$

where $X_{III} = X'_d + X_{TL1}$.

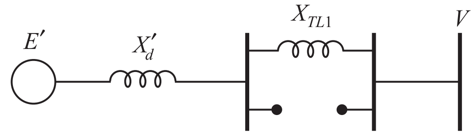


Figure 5.24 Postfault condition.

P_{e1}, P_{e2}, and P_{e3} are plotted in Figure 5.25.

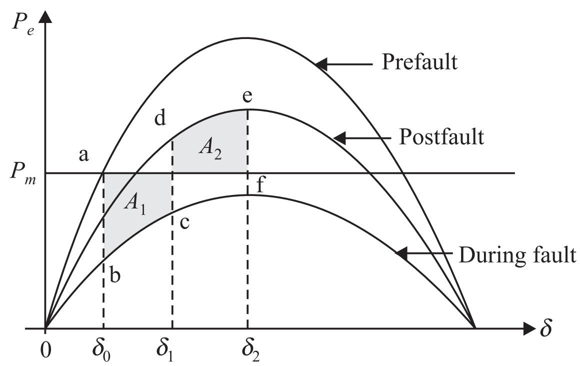


Figure 5.25 Power angle curves.

The accelerating area A_1 corresponding to δ_1 is less than area A_2, giving better chance for the stable operation. It is possible to find an area A_2 equal to area A_1. As δ_1 increases, area A_1 increases.

To find $A_1 = A_2$, δ_1 increases till $\delta_2 = \delta_{\max}$. This case of critical clearing angle is shown in Figure 5.26.

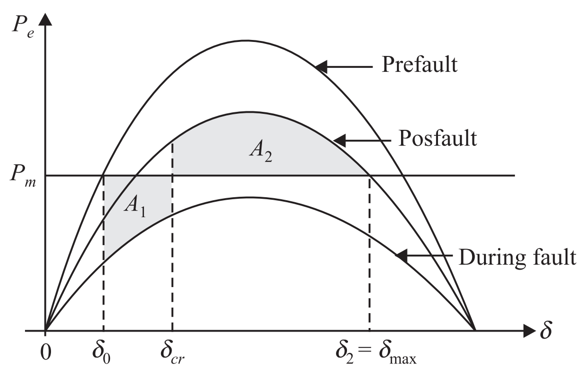


Figure 5.26 Power angle curves.

Applying the equal area criterion, we can write

$$\text{Area } A_1 = \text{Area } A_2$$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{\max 2} \sin\delta)\, d\delta = \int_{\delta_{cr}}^{\delta_{\max}} (P_{\max 3} \sin\delta - P_m)\, d\delta$$

where $\delta_{\max} = \pi - \sin^{-1}\left(\dfrac{P_m}{P_{\max 3}}\right)$

Integrating, we get

$$(P_m\delta + P_{\max 2}\cos\delta)\Big|_{\delta_0}^{\delta_{cr}} + (P_{\max 3}\cos\delta + P_m\delta)\Big|_{\delta_{cr}}^{\delta_{\max}} = 0$$

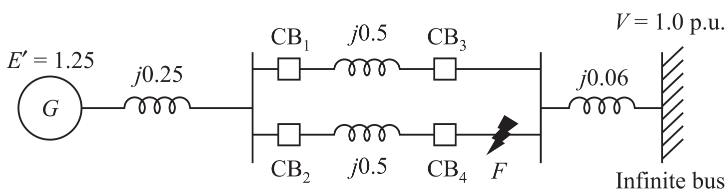
$$P_m(\delta_{cr} - \delta_0) + P_{\max 2}(\cos\delta_{cr} - \cos\delta_0) + P_m(\delta_{\max} - \delta_{cr}) + P_{\max 3}(\cos\delta_{\max} - \cos\delta_{cr}) = 0$$

$$\cos\delta_{cr} = \frac{P_m(\delta_{\max} - \delta_0) - P_{\max 2}\cos\delta_0 + P_{\max 3}\cos\delta_{\max}}{P_{\max 3} - P_{\max 2}}$$

The critical clearing angle can be calculated from the above equation. The angles in this equation are in radians. The equation modifies as below if the angles are in degrees.

$$\cos\delta_{cr} = \frac{P_m(\delta_{\max} - \delta_0)\dfrac{\pi}{180} - P_{\max 2}\cos\delta_0 + P_{\max 3}\cos\delta_{\max}}{P_{\max 3} - P_{\max 2}}$$

EXAMPLE 5.12 Given the circuit as shown in the figure below where a three-phase fault is applied on one end of a line near circuit breaker CB_4. Find the critical fault clearing angle for clearing the fault with simultaneous opening of breaker CB_2 and CB_4. The generator is delivering 1.0 p.u. MW at the instant preceding the fault. All the p.u. quantities are on the common MVA.



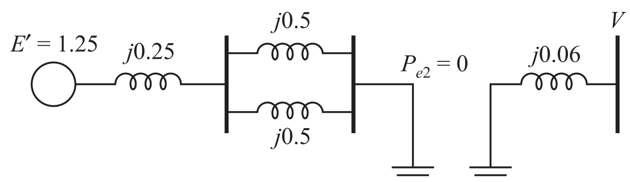
Solution:

Prefault condition Transfer reactance during prefault operation is

$$X_I = 0.25 + \frac{0.5 \times 0.5}{0.5 + 0.5} + 0.06 = 0.56$$

$$P_{e1} = \frac{|E'||V|}{X_I}\sin\delta = \frac{1.25 \times 1.0}{0.56}\sin\delta = 2.232\sin\delta$$

During fault condition The fault occurs at the end of the line 2 or near bus 2. Therefore during the short circuit fault the circuit separates by the circuit breaker for finding the transfer reactance as shown in the Figure below. During the clearing of fault, no power is transferred from the circuit, i.e. $P_{e2} = 0$.



Postfault condition With the opening of the faulted line, say by simultaneous opening of the circuit breakers CB_2 and CB_4, the postfault transfer reactance is

$$X_{III} = 0.25 + 0.5 + 0.06 = 0.81$$

$$P_{e3} = \frac{|E'||V|}{X_{III}}\sin\delta = \frac{1.25 \times 1.0}{0.81}\sin\delta = 1.543\sin\delta$$

The initial power angle δ_0 is calculated as

$$P_{m0} = P_{e0} = 1.0 = P_{\max 1}\sin\delta_0$$

$$\delta_0 = \sin^{-1}\left(\frac{1.0}{2.232}\right) = 26.32°$$

and

$$\delta_{\max} = 180° - \sin^{-1}\left(\frac{P_{m0}}{P_{\max 3}}\right)$$

$$\delta_{\max} = 180° - \sin^{-1}\left(\frac{1.0}{1.543}\right) = 139.6°$$

$$\cos\delta_{cr} = \frac{P_m(\delta_{\max} - \delta_0)\frac{\pi}{180} - P_{\max 2}\cos\delta_0 + P_{\max 3}\cos\delta_{\max}}{P_{\max 3} - P_{\max 2}}$$

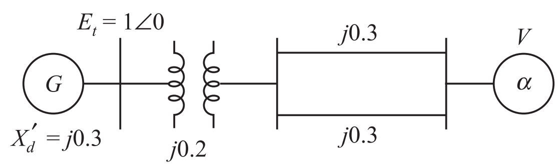
$$= \frac{1.0(139.6 - 26.62)\frac{\pi}{180} - 0 + 1.543\cos 139.6°}{1.543 - 0} = 0.51640$$

$$\delta_{cr} = 58.9°$$

EXAMPLE 5.13 A 2220 MVA, 24 kV and 60 Hz synchronous machine is connected to an infinite bus through transformer and double circuit transmission line, as shown in the following figure. The infinite bus voltage $V = 1.0$ p.u. The direct axis transient reactance of the machine is 0.30 p.u., the transformer reactance is 0.20 p.u., and the reactance of each transmission line is 0.3 p.u., all to a base of the rating of the synchronous machine. Initially, the machine is delivering 0.8 p.u. real power and reactive power is 0.074 p.u. with a terminal voltage of 1.0 p.u. The inertia constant $H = 5$ MJ/MVA. All resistances are neglected.

(i) A temporary three-phase fault occurs at the sending end of one of the lines. When the fault is cleared, both lines are intact. Determine the critical clearing angle and the critical fault clearing time.

(ii) A three-phase fault occurs at the middle of one of the lines, fault is cleared, and the faulted line is isolated. Determine the critical clearing angle.



Solution:

Given: infinite bus voltage V = 1.0 p.u.

terminal voltage of generator E_t =1.0 p.u.

$X'_d = j0.3$

$P_{m0} = P_{e0} = 0.8$

The current flowing into the infinite bus is

$$I = \frac{S^*}{V^*} = \frac{0.8 - j0.074}{1.0} = 0.8 - j0.074$$

The transfer reactance between the internal voltage and the infinite bus before fault is

$$X_I = 0.3 + 0.2 + \frac{0.3 \times 0.3}{0.3 + 0.3} = 0.65$$

$$E' = V + jX_I I = 1.0 + j(0.65)\,(0.8 - j0.074) = 1.17\angle 26.387°$$

(i) Three-phase fault occurs at the sending end of one of the lines

Prefault condition

$$X_I = 0.3 + 0.2 + \frac{0.3 \times 0.3}{0.3 + 0.3} = 0.65$$

$$P_{e1} = \frac{|E'||V|}{X_I}\sin\delta = \frac{1.17 \times 1.0}{0.65}\sin\delta = 1.8\sin\delta$$

During the fault condition: $P_{e2} = 0$

Postfault condition Since both the lines are intact when the fault is cleared therefore the power angle equation of prefault and postfault are the same.

$$P_{e3} = \frac{|E'||V|}{X_I}\sin\delta = \frac{1.17 \times 1.0}{0.65}\sin\delta = 1.8\sin\delta$$

The initial power angle δ_0 is calculated as

$$P_{m0} = P_{e0} = 0.8 = P_{\max 1} \sin \delta_0$$

$$\delta_0 = \sin^{-1}\left(\frac{0.8}{1.8}\right) = 26.388° = 0.46055 \text{ rad}$$

and

$$\delta_{\max} = 180° - \sin^{-1}\left(\frac{P_{m0}}{P_{\max 3}}\right)$$

$$\delta_{\max} = 180° - \sin^{-1}\left(\frac{0.8}{1.8}\right) = 153.612°$$

$$\cos \delta_{cr} = \frac{P_m(\delta_{\max} - \delta_0)\frac{\pi}{180} - P_{\max 2} \cos \delta_0 + P_{\max 3} \cos \delta_{\max}}{P_{\max 3} - P_{\max 2}}$$

$$= \frac{0.8(153.612 - 26.388)\frac{\pi}{180} - 0 + 1.8 \cos 153.612°}{1.8 - 0} = 1.48 \text{ rad}$$

$$\delta_{cr} = 84.775°$$

The critical clearing time

$$t_c = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m}}$$

$$t_c = \sqrt{\frac{2 \times 5(1.48 - 0.46055)}{\pi \times 60 \times 0.8}} = 0.26 \text{ s}$$

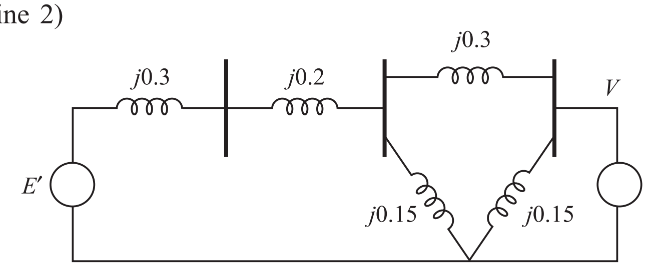
(ii) Three-phase fault occurs at the middle of one of the lines

Prefault condition From the reactance diagram,

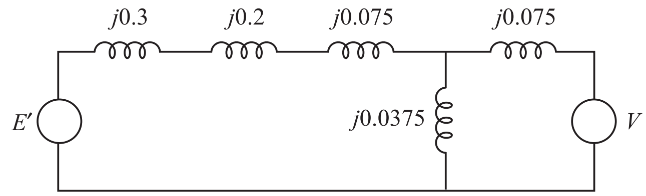
$$X_T = X_I = 0.3 + 0.2 + \left(\frac{0.3 \times 0.3}{0.3 + 0.3}\right) = 0.65$$

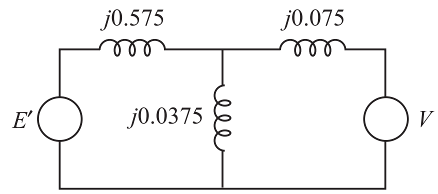
$$P_{e1} = \frac{|E'||V|}{X_I} \sin \delta = \frac{1.17 \times 1.0}{0.65} \sin \delta = 1.8 \sin \delta$$

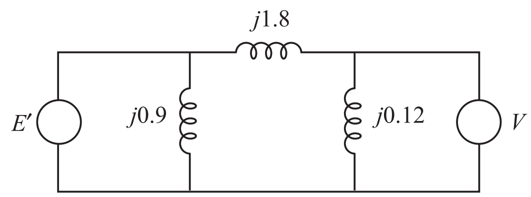
During the fault condition (fault at the middle of the transmission line 2)



Using delta to star conversion, the circuit becomes





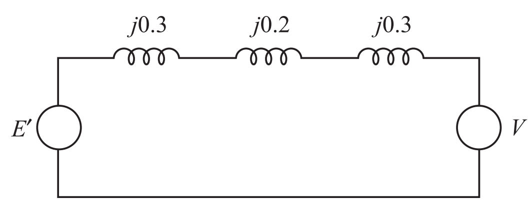


From the reactance diagram,

$$X_T = X_{II} = 1.8$$

$$P_{e2} = \frac{|E'||V|}{X_{II}}\sin\delta = \frac{1.17\times 1.0}{1.8}\sin\delta = 0.65\sin\delta$$

Postfault condition



From the reactance diagram,

$$X_T = X_{III} = 0.8$$

$$P_{e3} = \frac{|E'||V|}{X_{III}}\sin\delta = \frac{1.17\times 1.0}{0.8}\sin\delta = 1.4625\sin\delta$$

The initial power angle δ_0 is calculated as

$$P_{m0} = P_{e0} = 0.8 = P_{\max 1}\sin\delta_0$$

$$\delta_0 = \sin^{-1}\left(\frac{0.8}{1.8}\right) = 26.388° = 0.46055 \text{ rad}$$

and

$$\delta_{max} = 180° - \sin^{-1}\left(\frac{P_{m0}}{P_{max\,3}}\right)$$

$$\delta_{max} = 180° - \sin^{-1}\left(\frac{0.8}{1.4625}\right) = 146.838° = 2.5628 \text{ rad}$$

$$\cos\delta_{cr} = \frac{P_m(\delta_{max} - \delta_0)\dfrac{\pi}{180} - P_{max\,2}\cos\delta_0 + P_{max\,3}\cos\delta_{max}}{P_{max\,3} - P_{max\,2}}$$

$$= \frac{0.8(146.838 - 26.388)\dfrac{\pi}{180} - 0.65\cos 26.388° + 1.4625\cos 146.838°}{1.4625 - 0.65}$$

$$= -0.15356 \text{ rad}$$

$$\delta_{cr} = 98.834°$$

5.8 Solution of Swing Equation

The swing equation, governing the motion of each machine of a system, is

$$M\frac{d^2\delta}{dt^2} = P_a$$

where δ – displacement angle of rotor with respect to a reference axis rotating at normal speed

M – inertia constant of machine

P_a – accelerating power (difference between the mechanical input and output after correcting the losses)

t – time.

The solution of swing equation can be done by applying the following methods.

1. Step by step method.
2. Euler's method.
3. Modified Euler's method.
4. Runge–Kutta method.

The step by step method (point by point method) is the most feasible and widely used way of solving the swing equations. By this method, a good accuracy can be attained and it is also easy to compute δ.

In the step by step method, one or more variables are assumed constant throughout the short interval of time Δt, so that as a result of the assumptions made, the equations can be solved for the changes in the other variables during the same time interval. Then from the values of the other variables at the end of the interval, the new values can be calculated for the variables which were assumed constant. These new values are then used in the next time interval.

5.8.1 Step by Step Method-I

It consists of two processes. These are to be carried out alternately.

1. Assume accelerating power P_a to be constant and compute the angular position and if necessary, the angular velocity (speed) at the end of the time interval from the knowledge of the position and speeds at the beginning of the interval.
2. Then from the angular positions and speed, the accelerating power of each machine is to be calculated. This P_a will be kept constant for the next time interval and the procedure is to be repeated till the required final time is reached.

The swing equation is given by

$$M \frac{d^2\delta}{dt^2} = P_a$$

The above equation is written as,

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

Integrating with respect to t,

$$\frac{d\delta}{dt} = \omega = \omega_0 + \frac{P_a}{M} t \tag{5.27}$$

Integrating once again,

$$\delta = \delta + \omega_0 t + \frac{P_a}{M} \cdot \frac{t^2}{2} \tag{5.28}$$

Equations (5.27) and (5.28) respectively provide the speed of the machine (ω) and the angular displacement (δ) of the machine with respect to a reference axis rotating at synchronous speed.

δ_0 and ω_0 are the values of δ and ω at the beginning of the interval. These equations hold for any instant of time t during the interval in which P_a is constant.

We are in the need of δ and ω at the end of the interval.

Let n denote the quantities at the end of the nth interval and n – 1 denote the quantities at the end of the (n – 1)th interval, which is the beginning of the nth interval. Δt is the length of the interval.

Putting Δt in place of t in Eqs. (5.27) and (5.28) and using the appropriate subscripts, we obtain the speed and angle at the end of the nth interval as

$$\begin{aligned} \omega_n &= \omega_{n-1} + \frac{\Delta t}{M} P_a(n-1) \\ \delta_n &= \delta_{n-1} + \Delta t\, \omega_{n-1} + \frac{\Delta t}{2M} P_a(n-1) \end{aligned} \tag{5.29}$$

The increments of speed and angle during the *n*th interval are

$$\begin{aligned}\Delta\omega_n &= \omega_n - \omega_{n-1} = \frac{\Delta t}{M} P_a(n-1) \\ \Delta\delta_n &= \delta_n - \delta_{n-1} = \Delta t\, \omega_{n-1} + \frac{\Delta t^2}{2M} P_a(n-1)\end{aligned} \tag{5.30}$$

Equation (5.29) or (5.30) is suitable for step by step calculation.

If we are interested only in the angular position and not in the speed, ω_{n-1} can be eliminated from Eqs. (5.29) and (5.30).

For the preceding interval, we can write

$$\delta_{n-1} = \delta_{n-2} + \Delta t\, \omega_{n-2} + \frac{(\Delta t)^2}{2M} P_{a(n-2)} \tag{5.31}$$

Equation (5.29)–(5.31) gives

$$\delta_n - \delta_{n-1} = \delta_{n-1} - \delta_{n-2} + (\omega_{n-1} - \omega_{n-2})\Delta t + \frac{\Delta t^2}{2M}(P_{a\,(n-1)} - P_{a\,(n-2)})$$

Putting $\Delta\delta_n = \delta_n - \delta_{n-1}$; $\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2}$
$\Delta\omega_{n-1} = \omega_{n-1} - \omega_{n-2}$

$$\Delta\delta_n = \Delta\delta_{n-1} + \Delta\omega_{n-1} \cdot \Delta t + \frac{\Delta t^2}{2M}(P_{a\,(n-1)} - P_{a\,(n-2)}) \tag{5.32}$$

But from Eq. (5.30)

$$\Delta\omega_{n-1} = \frac{\Delta t}{M} P_{a\,(n-2)}$$

Substituting in Eq. (5.32)

$$\begin{aligned}\Delta\delta_n &= \Delta\delta_{n-1} + \frac{\Delta t}{M} \cdot P_{a(n-2)} \cdot \Delta t + \frac{\Delta t^2}{2M}(P_{a\,(n-1)} - P_{a\,(n-2)}) \\ &= \Delta\delta_{n-1} + \frac{\Delta t^2}{2M}(P_{a\,(n-1)} - P_{a\,(n-2)})\end{aligned}$$

This equation gives the increment in angle during any interval in terms of the increment for the previous interval.

Time interval Δt should be short enough to give the required accuracy. If it is too short, it will increase the number of calculations to plot a swing curve, and thus it provides accuracy.

Limitations of step by step method-I

The acceleration during each interval of time Δt is constant at the value corresponding to the beginning of the interval.

Figure 5.27 shows the true variation of acceleration (α) as a function of time and the assumed variation of the above method.

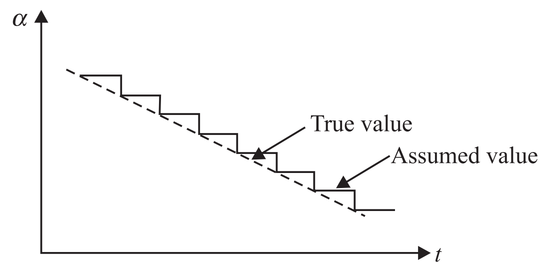


Figure 5.27 Variation of acceleration (α) as a function of time.

EXAMPLE 5.14 Consider a 60 Hz machine for which H = 2.7 MJ/MVA and it is initially operating in steady state with input and output of 1 p.u. and an angular displacement of 45 electrical degree with respect to an infinite bus bar. Upon occurrence of a fault, assume that the input remains constant and the output is given by $P_e = \delta/90°$. Calculate and plot swing curve by step by step method I. Using the time interval Δt = 0.05 s. Up to t = 1 s.

Solution:

Given P_i = 1 p.u.; S = 1 p.u.

$$P_e = \frac{\delta}{90°}$$

Also
$$P_a = P_i - P_e = 1 - \frac{\delta}{90°}$$

$\therefore$
$$P_a = 1 - 0.0111\delta$$

$$M = \frac{SH}{180f} = \frac{2.7}{180 \times 60} = 2.5 \times 10^{-4}$$

We have, by method I

$$\Delta\omega_n = \frac{\Delta t}{M} P_{a(n-1)}$$

$$\Delta\delta_n = \Delta t \cdot \omega_{n-1} + \frac{\Delta t^2}{2M} P_{a(n-1)}$$

Substituting Δt and M in the above two equations

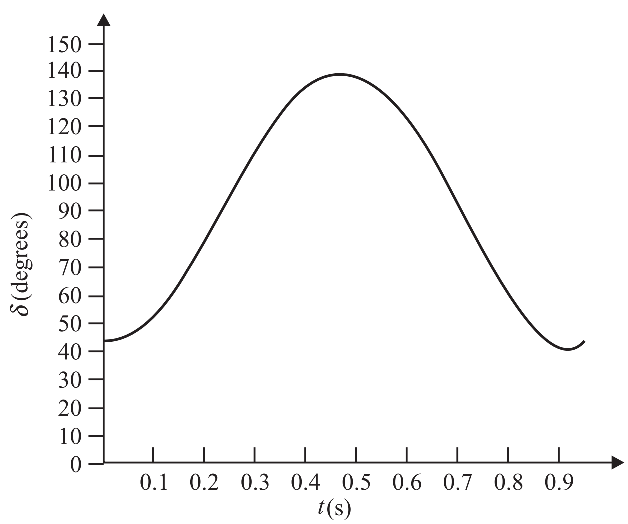
$$\Delta\omega_n = \frac{0.05}{2.5 \times 10^{-4}} P_{a(n-1)}; \qquad \delta_n = \Delta t \cdot \omega_{n-1} + \frac{\Delta t^2}{2M} P_{a\,(n-1)}$$

$$\Delta\omega_n = 200 P_{a\,(n-1)} \qquad \Delta\delta_n = 0.05\,\omega_{n-1} + 5P_{a\,(n-1)}$$

t s	P_e	P_a	$\Delta\omega$	ω	0.05 ω	5P_a	$\Delta\delta$ deg.	δ deg.
0+	0.5	0.5	100	0	0	2.5	2.5	45
0.05	0.528	0.472	94.55	100	5	2.36	7.36	47.5
0.1	0.6089	0.391	78.2	194.55	9.7275	1.955	11.6825	54.86
0.15	0.7386	0.2614	52.2757	272.75	13.6375	1.307	14.9445	66.5425
0.2	0.9045	0.0955	19.1	325.0257	16.2513	0.4775	16.7288	81.487
0.25	1.0902	–0.0902	–18.04	344.13	17.21	–0.45	16.76	98.2158
0.3	1.28	–0.28	–55.24	326.09	16.3	–1.4	14.9	114.97
0.35	1.441	–0.44	–88.31	270.85	13.54	–2.2	11.34	129.87
0.4	1.57	–0.57	–113.49	182.54	9.13	–2.85	6.28	141.21
0.45	1.64	–0.64	–127.43	69.05	3.45	–3.2	0.25	147.49
0.5	1.64	–0.64	–127.98	–58.38	–2.92	–3.2	–6.12	147.74
0.55	1.57	–0.57	–114.4	–186.36	–9.32	–2.85	–12.17	141.62
0.6	1.44	–0.44	–88	–300.76	–15.04	–2.2	–17.24	129.45
0.65	1.25	–0.25	–49.11	–388.76	–19.44	–1.25	–20.69	112.21
0.7	1.02	–0.02	–3.17	–437.87	–21.89	–0.1	–21.99	91.52
0.75	0.77	0.23	46	–441.04	–22.05	1.15	–20.9	69.53
0.8	0.54	0.46	92	–395.04	–19.75	2.3	–17.45	48.63
0.85	0.35	0.65	130.75	–303.04	–15.15	3.25	–11.9	31.18
0.9	0.21	0.79	157.2	–172.26	–8.61	3.93	–4.68	19.28
0.95	0.16	0.84	167.59	–15.06	–0.75	4.2	3.45	14.6
1	0.2	0.8	159.94	152.53	7.63	4	11.63	18.05
								29.68

The variation of δ with time t is plotted as shown in the figure below

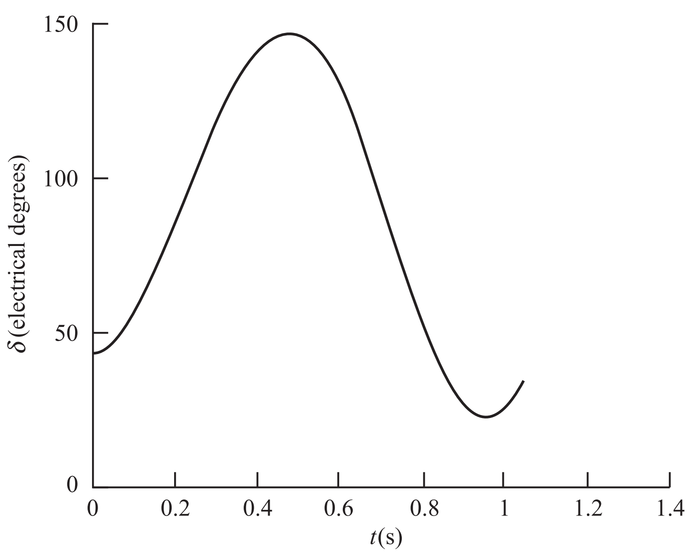
From the swing curve, we see that the value of δ increases and then decreases, therefore the system is stable. A more accurate approach would be to assume the average value of acceleration over the interval Δt, by using the value of acceleration at the middle of the interval. It is adopted in step by step method II.



MATLAB program—solution of swing equation using step by step method – I

```
Del_delta=0; i=1;
f=input('Enter the frequency :');
H=input('Enter the value of inertia constant :');
delta=input('Enter initial displacement angle :');
Pi=input('Enter initial steady state power :');
M=H/(180*f); y=0.05/M;yy=0.05^2/(2*M);Pu=delta/90;
del_int=delta*pi/180; Pa=1-Pu;
disp('Time Pe Pa 5*Pa Del_ang Del')
for t=0:.05:1.05
 if t==0
 w=0;
 del_w=y*Pa;
 del_del=0.05*w+yy*Pa;
 fprintf('%g-',t);
 disp([Pi Pa yy*Pa del_del delta]);
 fprintf('%g+',t);
 disp([Pu Pa yy*Pa del_del delta])
 else
 w=w+del_w;
 delta=delta+del_del;
 Pu=delta/90;
 Pa=1-Pu;
 del_w=y*Pa;
 del_del=0.05*w+yy*Pa;
 fprintf('%g',t);
 disp([Pu Pa yy*Pa del_del delta])
 end
 time(i)=t;
 del(i)=delta;
 delta=delta+Del_delta;
 i=i+1;
end
plot(time,del);
title('SWING CURVE');
xlabel('t, sec');
ylabel('delta, elec. deg');
Results:
Enter the value of inertia constant: 2.7
Enter the initial displacement angle: 45
Enter the initial steady state power: 1
```

Time	*Pe*	*Pa*	5**Pa*	Del_ang	Del
0+	0.5	0.5	2.5	2.5	45
0.05	0.52778	0.47222	2.3611	7.3611	47.5
0.1	0.60957	0.39043	1.9522	11.674	54.861
0.15	0.73928	0.26072	1.3036	14.93	66.535
0.2	0.90517	0.09483	0.47413	16.708	81.466
0.25	1.0908	–0.0908	–0.4541	16.728	98.173
0.3	1.2767	–0.2767	–1.3834	14.89	114.9
0.35	1.4421	–0.4421	–2.2107	11.296	129.79
0.4	1.5676	–0.5677	–2.8382	6.2475	141.09
0.45	1.6371	–0.6371	–3.1853	0.22392	147.34
0.5	1.6395	–0.6396	–3.1977	–6.1591	147.56
0.55	1.5711	–0.5711	–2.8556	–12.212	141.4
0.6	1.4354	–0.4354	–2.1771	–17.245	129.19
0.65	1.2438	–0.2438	–1.219	–20.641	111.94
0.7	1.0145	–0.0145	–0.0723	–21.933	91.301
0.75	0.77076	0.22924	1.1462	–20.859	69.369
0.8	0.539	0.461	2.305	–17.408	48.51
0.85	0.34558	0.65442	3.2721	–11.831	31.102
0.9	0.21413	0.78587	3.9293	–4.6291	19.272
0.95	0.1627	0.8373	4.1865	3.4868	14.643
1	0.20144	0.79856	3.9928	11.666	18.13

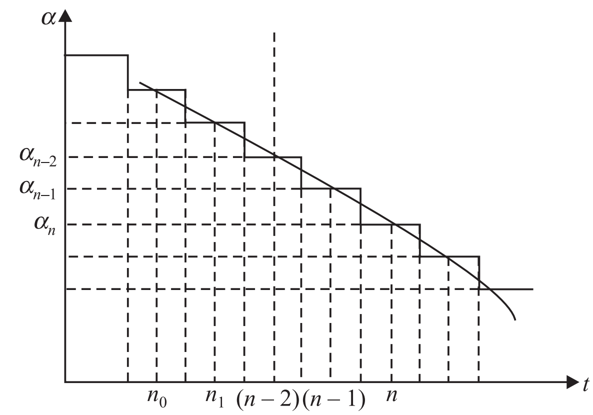


Swing curve

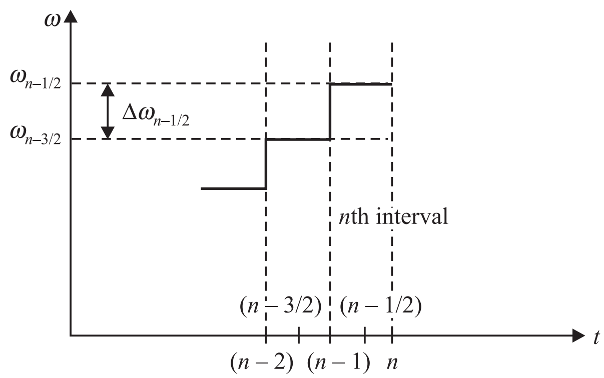
5.8.2 Step by Step Method-II

Acceleration is assumed constant from the middle of one interval to the middle of the next interval.

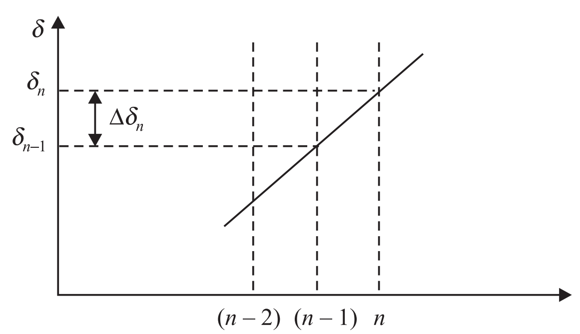
For example, consider Figure 5.28(a) where α_{n-1} is the equivalent constant value of acceleration from $t = (n - (3/2))\Delta t$ to $t = (n - (1/2))\Delta t$.



(a) α constant from the middle of one interval to the middle of the next



(b) Angular velocity (ω) constant during the interval



(c) Displacement angle δ

Figure 5.28 Step by step method-II—variation of α, ω and δ.

α_n is constant from $t = (n - (1/2))\Delta t$ to $t = (n + (1/2))\Delta t$ and so on. In the region of constant acceleration α_{n-1}, there is an increment of angular velocity from $\omega_{n-3/2}$ to $\Delta\omega_{n-1/2}$, i.e. [from Figure 5.28(b)]

$$\omega_{n-1/2} = \omega_{n-3/2} + \Delta\omega_{n-1/2} \tag{5.33}$$

where $\Delta\omega_{n-1/2}$ is the increment in angular velocity over a time interval Δt, due to acceleration α_{n-1}.

$$\Delta\omega_{n-1/2} = \alpha_{n-1} \cdot \Delta t \tag{5.34}$$

As

$$\alpha_{n-1} = \frac{d^2\delta_{n-1}}{dt^2} = \frac{P_{a\,(n-1)}}{M}$$

Thus,

$$\Delta\omega_{n-1/2} = \frac{P_{a\,(n-1)}}{M} \cdot \Delta t \tag{5.35}$$

Again, the angular velocity, $\omega_{n-1/2}$ remains constant for $t = (n - 1)\Delta t$, during the nth interval. From Figure 5.28(c), the displacement angle δ_{n-1} increases to δ_n over this interval by an amount $\Delta\delta_n$.

$$\delta_n = \delta_{n-1} + \Delta\delta_n \tag{5.36}$$

$$\Delta\delta_n = \omega_{n-1/2} \cdot \Delta t \tag{5.37}$$

Using Eq. (5.33)

$$\omega_{n-1/2} = \omega_{n-3/2} + \Delta\omega_{n-1/2}$$

Hence

$$\begin{aligned}\Delta\delta_n &= (\omega_{n-3/2} + \Delta\omega_{n-1/2})\Delta t \\ &= \omega_{n-3/2}\,\Delta t + \alpha_{n-1}\Delta t^2 \qquad \text{by using Eq. (5.34)}\end{aligned} \tag{5.38}$$

From Eq. (5.37)

$$\Delta\delta_{n-1} = (\omega_{n-3/2})\Delta t \tag{5.39}$$

Substituting Eq. (5.39) in Eq. (5.38), we get

$$\Delta\delta_n = \Delta\delta_{n-1} + \alpha_{n-1}\Delta t^2$$

Substituting $\alpha_{n-1} = P_{a(n-1)}/M$, we get

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{P_{a(n-1)}}{M}\Delta t^2 \tag{5.40}$$

In the method II, it is not necessary to calculate ω, unless it is specifically required.

Using Eq. (5.40), P_a is to be found step by step over a number of intervals of time, from which the increment in displacement angle can be calculated as the inertia constant is known.

To begin with, the power output at $t = 0^-$ and 0^+ are averaged out and the average value of P_a is determined. Then $\Delta\delta$ is calculated from the value during the preceding interval.

EXAMPLE 5.15 Consider a 60 Hz machine for which $H = 2.7$ MJ/MVA and it is initially operating in steady state with input and output of 1 p.u. and an angular displacement of 45 electrical degree with respect to an infinite bus bar. Upon occurrence of a fault, assume that the input remains constant and the output is given by $P_e = \delta/90°$. Calculate and plot the swing curve by the step-by-step method II. Using the time interval $\Delta t = 0.05$ s. Up to $t = 1$ s. Step-by-step method-II using the time interval $\Delta t = 0.05$ s and upto $t = 1$ s.

Solution:

$H = 2.7$ MJ/MVA

At time $t = 0^-$, $P_m = P_e = 1$ p.u.

At time $t = 0^+$, $P_m = 1$ p.u., $P_e = \delta/90°$

$$M = \frac{SH}{180f} = 2.5 \times 10^{-4} \text{ p.u.}$$

$$P_a = 1 - \frac{\delta}{90°}$$

$$\frac{\Delta t^2}{M} = \frac{0.05^2}{2.5 \times 10^{-4}} = 10$$

Now the equation for $\Delta\delta_n$ is given by

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{\Delta t^2}{M} P_{a\,(n-1)}$$

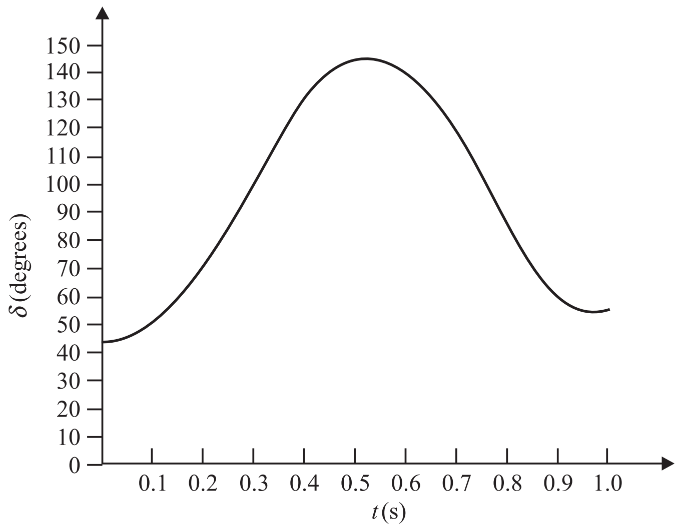
$$= \Delta\delta_{n-1} + 10P_{a(n-1)}$$

t(s)	P_e	P_a	$10P_a$	$\Delta\delta$ deg.	δ deg.
0^-	1	0	—	—	—
0^+	0.5	0.5	—	—	45°
0_{av}	—	0.25	2.5	—	—
				2.5	
0.05	0.5277	0.472	4.72	—	47.5
				7.22	
0.1	0.608	0.392	3.92	—	54.72
				11.14	
0.15	0.732	0.2682	2.682	—	65.86
				13.822	

(*Contd...*)

(*Contd...*)

t(s)	P_e	P_a	$10P_a$	$\Delta\delta$ deg.	δ deg.
0.2	0.8854	0.1146	1.1464	—	79.68
				14.9764	
0.25	1.05174	–0.0517	–0.5173	—	94.6564
				14.459	
0.3	1.212	–0.2124	–2.1239	—	109.115
				12.335	
0.35	1.3495	–0.3495	–3.495	—	121.45
				8.84	
0.4	1.448	–0.448	–4.48	—	130.29
				4.36	
0.45	1.496	–0.496	–4.96	—	134.65
				–0.6	
0.5	1.489	–0.489	–4.89	—	134.05
				–5.49	
0.55	1.428	–0.428	–4.28	—	128.56
				–9.77	
0.6	1.32	–0.32	–3.2	—	118.79
				–12.97	
0.65	1.176	–1.1758	–1.757	—	105.82
				–14.727	
0.7	1.012	–0.012	–0.12	—	91.093
				–14.877	
0.75	0.847	0.157	1.53	—	76.216
				–13.347	
0.8	0.699	0.301	3.01	—	62.869
				–10.337	
0.85	0.584	0.416	4.16	—	52.532
				–6.177	
0.9	0.515	0.485	4.85	—	46.355
				–1.327	
0.95	0.5	0.5	5		45.028
				3.673	
1					48.701



From the swing curve, we see that the value of δ increases and then decreases, which makes the system stable.

Note: In the method-II, the average of the accelerating powers was calculated from the prefault (0^-) and during the fault (0^+) powers, and used in the further iterations. Similarly, if the fault is cleared by some means at any intermediate time t, the same method of calculation of average power is to be adopted for during the fault (t^-) and the postfault (t^+) time periods, and these values are to be used in further calculations.

MATLAB program—Solution of swing equation using step by step method–II

```
Del_delta=0; i=1;
f=input('Enter the frequency :');
H=input('Enter the value of inertia constant :');
delta=input('Enter initial displacement angle :');
Pi=input('Enter initial steady state power :');
M=H/(180*f); y=0.05/M;yy=0.05^2/(M);Pu=delta/90;
del_int=delta*pi/180; Pa=1-Pu;
disp('Time Pe Pa 10*Pa Del_ang Del')
for t=0:.05:1.05
 if t==0
 w=0;
 del_w=y*Pa;
 del_del=0.05*w+yy*Pa;
 fprintf('%g-',t);
 disp([Pi Pa del_w del_del delta]);
 fprintf('%g+',t);
```

```
    disp([Pu Pa del_w del_del delta])
    Pa=Pa/2;
    del_w=y*Pa;
    del_del=0.05*w+yy*Pa;
    fprintf('%gavg',t);
    disp([Pu Pa yy*Pa del_del delta])
  else
    w=w+del_w;
    delta=delta+del_del;
    Pu=delta/90;
    Pa=1-Pu;
    del_w=y*Pa;
    del_del=0.05*w+yy*Pa;
    fprintf('%g',t);
    disp([Pu Pa yy*Pa del_del delta])
  end
  time(i)=t;
  del(i)=delta;
  delta=delta+Del_delta;
  i=i+1;
end
plot(time,del);
title('SWING CURVE');
xlabel('t, sec');
ylabel('delta, elec. deg');
```

Results:

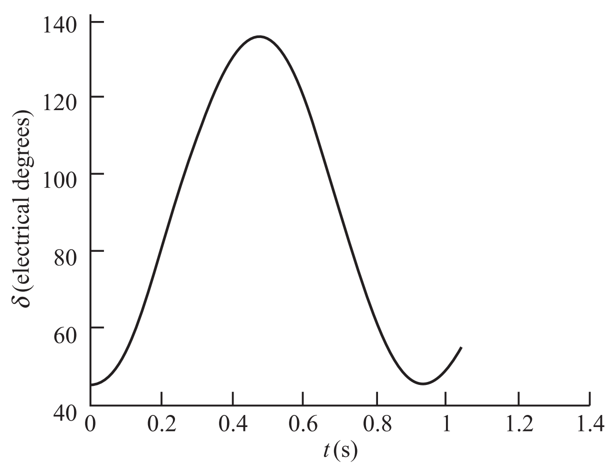
Enter the value of inertia constant : 2.7
Enter the initial displacement angle : 45
Enter the initial steady state power : 1

Time	*Pe*	*Pa*	10**Pa*	Del_ang	Del
0–	1	0.5	—	—	45
0+	0.5	0.5	—	—	45
0avg	—	0.25	2.5	—	45
0.05	0.52778	0.47222	4.7222	7.2222	47.5
0.1	0.60802	0.39198	3.9198	11.142	54.722
0.15	0.73182	0.26818	2.6818	13.824	65.864
0.2	0.88542	0.11458	1.1458	14.97	79.688
0.25	1.0517	–0.0517	–0.5174	14.452	94.657
0.3	1.2123	–0.2123	–2.1233	12.329	109.11
0.35	1.3493	–0.3493	–3.4931	8.8356	121.44
0.4	1.4475	–0.4474	–4.4749	4.3607	130.27

(*Contd...*)

(*Contd...*)

Time	P_e	P_a	$10*P_a$	Del_ang	Del
0.45	1.4959	–0.4959	–4.9594	–0.5986	134.63
0.5	1.4893	–0.4892	–4.8929	–5.4915	134.04
0.55	1.4283	–0.4282	–4.2827	–9.7742	128.54
0.6	1.3197	–0.3196	–3.1967	–12.971	118.77
0.65	1.1755	–0.1755	–1.7555	–14.726	105.8
0.7	1.0119	–0.0119	–0.1192	–14.846	91.073
0.75	0.84697	0.15303	1.5303	–13.315	76.227
0.8	0.69902	0.30098	3.0098	–10.306	62.912
0.85	0.58452	0.41548	4.1548	–6.1507	52.606
0.9	0.51618	0.48382	4.8382	–1.3124	46.456
0.95	0.50159	0.49841	4.9841	3.6717	45.143
1	0.54239	0.45761	4.5761	8.2478	48.815



Swing curve

From the swing curve, the value of δ increases and then decreases, which makes the system stable.

EXAMPLE 5.16 A 20 MVA, 3-ϕ, 50 Hz generator delivers rated power at unity power factor via a double circuit transmission line to an infinite bus bar. The generator unit has a kinetic energy of 2.5 MJ/MVA at rated speed. Its $X'_d = 0.3$ p.u. The transformer circuits have negligible resistances and each has a reactance of 0.3 p.u. on a 20 MVA base. The voltage behind the transient reactance is 1.05 p.u. and the voltage of the infinite bus is 1 p.u. A 3-ϕ short circuit occurs at the middle of one of the transformer circuits. It involves ground (a) What is the initial displacement angle of the machine? The fault is cleared in 0.4 s by simultaneous opening of CBs at both ends of the faulted transmission line. (b) Calculate and plot the swing curve for the system and ascertain whether the system is stable or not.

Take Δt = 0.05 s and t_{max} = 1 s.

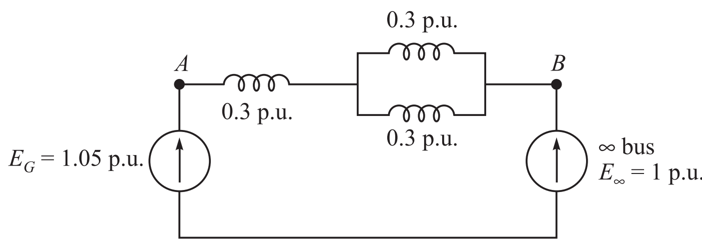
Solution:

$$M = \frac{SH}{180f} = \frac{1\times 2.5}{180\times 50} = 2.778\times 10^{-4}$$

$$P_i = \frac{\text{MW}}{\text{MVA}} = \frac{20\times 1}{20} = \frac{20}{20} = 1\text{ p.u.}$$

The power angle equation for the three conditions should be taken into account.

1. Prefault; 2. During fault; 3. Postfault



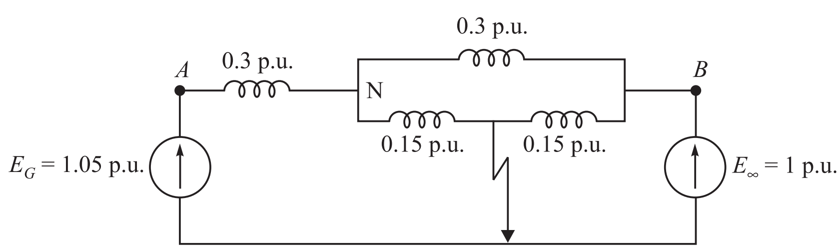
Prefault condition

$$X_1 = X_{AB} = 0.3 + \frac{0.3\times 0.3}{0.3+0.3} = 0.3 + 0.15$$

$$X_1 = 0.45\text{ p.u.}$$

During fault

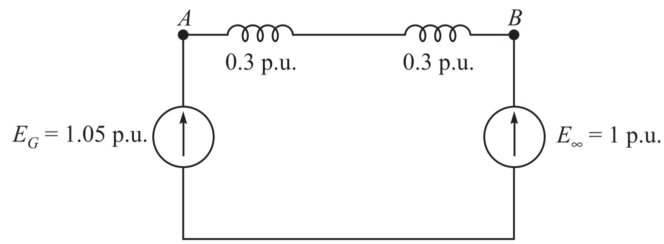
$X_2 = X_{AB}$ is obtained by converting the star connection to delta connection.



$$\therefore \qquad X_2 = X_{AB} = 0.3 + 0.3 + \frac{0.3\times 0.3}{0.15} = 0.6 + 0.6$$

$$X_2 = 1.2\text{ p.u.}$$

Postfault The faulty transmission line is made out of service by simultaneous opening of the circuit breakers at both ends and hence the faulty line is removed.



$$X_3 = X_{AB} = 0.3 + 0.3$$
$$= 0.6 \text{ p.u.}$$

Prefault power, $P_{\max 1} = \dfrac{1.05 \times 1}{0.45} = 2.333$ p.u.

During fault power, $P_{\max 2} = \dfrac{1.05 \times 1}{1.2} = 0.875$ p.u.

Postfault power, $P_{\max 3} = \dfrac{1.05 \times 1}{0.6} = 1.75$ p.u.

We know that

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{\Delta t^2}{M} P_{a(n-1)}$$

$$\therefore \quad \frac{\Delta t^2}{M} = \frac{(0.05)^2}{2.7778 \times 10^{-4}} = 9$$

$$\therefore \quad \Delta\delta_n = \Delta\delta_{n-1} + 9P_{a(n-1)}$$

$$P_i = 1 \text{ p.u.}$$

$$P_a = 1 - P_e$$

In general, $P_e = P_{\max} \sin\delta$

For the prefault condition

$$P_{\max 1} \sin\delta = P_m$$

$$2.333 \sin\delta = 1$$

$$\sin\delta = \frac{1}{2.333} = 0.4286$$

$$\therefore \quad \delta = 25.3808$$

For the during fault condition, $P_e = P_{\max 2} \sin\delta$

For the postfault condition, $P_e = P_{\max 3} \sin\delta$

The calculation of δ versus t is done in the following table.

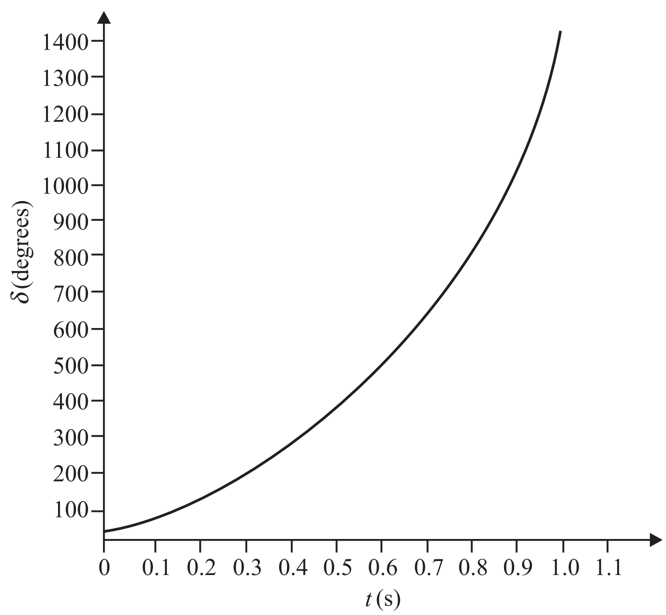
t (s)	**P_{max}**	**sin δ**	**P_e = P_{max} sin δ**	**P_a**	**$9P_a$**	**$\Delta\delta$ degree**	**δ degree**
0^-	2.333	0.4286	1	—	—	—	25.3808
0^+	0.875	0.4286	0.375	0.625	—	—	25.3808
0_{av}	—	—	—	0.3125	2.8125	—	—
						2.8125	
0.05	0.875	0.4724	0.4134	0.5866	5.2795	—	28.1933
						8.0920	
0.1	0.875	0.5918	0.5178	0.4822	4.3395	—	36.2853
						12.4315	
0.15	0.875	0.7515	0.6575	0.3425	3.0823	—	48.7168
						15.5138	
0.2	0.875	0.9006	0.7880	0.2120	1.9082	—	64.2306
						17.4220	
0.25	0.875	0.9894	0.8657	0.1343	1.2084	—	81.6526
						18.6304	
0.3	0.875	0.9839	0.8609	0.1391	1.2515	—	100.2830
						19.8819	
0.35	0.875	0.8646	0.7565	0.2435	2.1914	—	120.1649
						22.0733	
0.4^-	0.875	0.6124	0.5358	—	—	—	142.2382
0.4^+	1.75	0.6124	1.0717	—	—	—	142.2382
0.4_{av}	—	—	0.8038	0.1962	1.7658	—	
						23.8391	
0.45	1.75	0.2406	0.4211	0.5789	5.2104		166.0773
						29.0495	
0.5	1.75	–0.2610	–0.4567	1.4567	13.1100		195.1268
						42.1595	
0.55	1.75	–0.8414	–1.4724	2.4724	22.2518		237.28
						64.4113	
0.6	1.75	–0.8508	–1.489	2.489	22.4006		301.6976
						86.8119	
0.65	1.75	0.4773	0.8353	0.1647	1.4825		388.5095
						88.2944	
0.7	1.75	0.8926	1.5620	–0.5620	–5.0577		476.8039
						83.2367	
0.75	1.75	–0.3427	–0.5997	1.5997	14.3973		560.0406
						97.634	
0.8	1.75	–0.8856	–1.5498	2.5498	22.9482		657.67
						120.5822	

(*Contd...*)

(*Contd...*)

t (s)	P_{max}	$\sin\delta$	$P_e = P_{max}\sin\delta$	P_a	$9P_a$	$\Delta\delta$ degree	δ degree
0.85	1.75	0.8504	1.4882	–0.4882	–4.394		778.2568
						116.1882	
0.9	1.75	0.0968	0.1694	0.8306	7.4754		894.445
						123.6636	
0.95	1.75	–0.8821	–1.5436	2.5436	22.8924		1018.1086
						146.556	
1.0	1.75	0.9957	1.7424	–0.7424	–6.6818		1164.6646
						139.8742	
1.05							1304.5388

The variation of δ with time t is plotted as shown below.



(a) Initial displacement angle $\delta_0 = 25.3808$.

(b) From the swing curve, the displacement angle δ goes on increasing. Hence the system is unstable.

MATLAB program—solution of swing equation by considering critical clearing time using step by step method–II

```
clc;
clear;format short g;
Pm=input('Enter the power limits for prefault,during
fault,postfault conditions :');
Del_delta=0; i=1;
```

```
f=input('Enter the frequency :');
tc=input('Enter the Fault Clearing Time :');
H=input('Enter the value of inertia constant :');
delta=input('Enter initial displacement angle :');
Pi=input('Enter initial steady state power :');
M=H/(180*f); y=0.05^2/M;
k1=Pm(2)/Pm(1);k2=Pm(3)/Pm(1);
del_int=delta*pi/180;
del_max=pi-asin(sin(del_int)/k2);
del_cri=acos(((del_max-del_int)*sin(del_int)-k1*cos(del_
int)+k2*cos(del_max))/(k2-k1));
cri_t=(2*M*(del_cri-del_int)/Pi)^(1/2);
del_cri=(del_cri*180)/pi;
disp('Time P_max Sin(del) Pe Pa 9*Pa Del_ang Delta\n')
for t=0:.05:1.05
 delta=delta*pi/180;
 if t==0
    Paminus=Pi-1;
    Paplus=Pi-Pm(2)*sin(delta);
    Paavg=(Paminus+Paplus)/2;Pma=(Pm(1)+Pm(2))/2;sd=sin(delta);
    Pa=Paavg;delta=(delta*180)/pi;
fprintf('%g-',t);
disp([Pm(1) sd Pi Paminus (y*Paminus) Del_delta delta]);
fprintf('%g+',t);
disp([Pm(2) sd Pm(2)*sd Paplus (y*Paplus) Del_delta delta])
fprintf('%gavg',t);
disp([Pma sd Pma*sd Pa (y*Pa) Del_delta delta])
end
 if t==tc
    Paminus=Pi-Pm(2)*sin(delta);
    Paplus=Pi-Pm(3)*sin(delta);
    Paavg=(Paminus+Paplus)/2;Pma=(Pm(3)+Pm(2))/2;sd=sin(delta);
    Pa=Paavg;delta=(delta*180)/pi;
fprintf('%g-',tc);
disp([Pm(2) sd Pm(2)*sd Paminus (y*Paminus) Del_delta delta])
fprintf('%g+',tc);
disp([Pm(3) sd Pm(3)*sd Paplus (y*Paplus) Del_delta delta])
fprintf('%gavg',tc);
disp([Pma sd Pma*sd Pa (y*Pa) Del_delta delta])
 end
 if t>0 && t<tc
    Pa=Pi-Pm(2)*sin(delta);sd=sin(delta);delta=(delta*180)/pi;
disp([t Pm(2) sd Pm(2)*sd Pa (y*Pa) Del_delta delta])
 end
```

```
  if (t>tc)
     Pa=Pi-Pm(3)*sin(delta);sd=sin(delta);delta=(delta*180)/pi;
disp([t Pm(3) sd Pm(3)*sd Pa (y*Pa) Del_delta delta])
  end
  Del_delta=Del_delta+(y*Pa);
  time(i)=t;
  del(i)=delta;
  delta=delta+Del_delta;
  i=i+1;
end
critical_clearing_angle=del_cri
critical_clearing_time=cri_t
plot(time,del);
title('SWING CURVE');
xlabel('t, sec');
ylabel('delta, elec. deg');
```

Results:

Enter the power limits for prefault, during fault, and postfault conditions: [2.333 .875 1.75]
Enter the frequency: 50
Enter the fault clearing time: 0.4
Enter the value of inertia constant: 2.5
Enter the initial displacement angle: 25.3808
Enter the initial steady state power: 1.

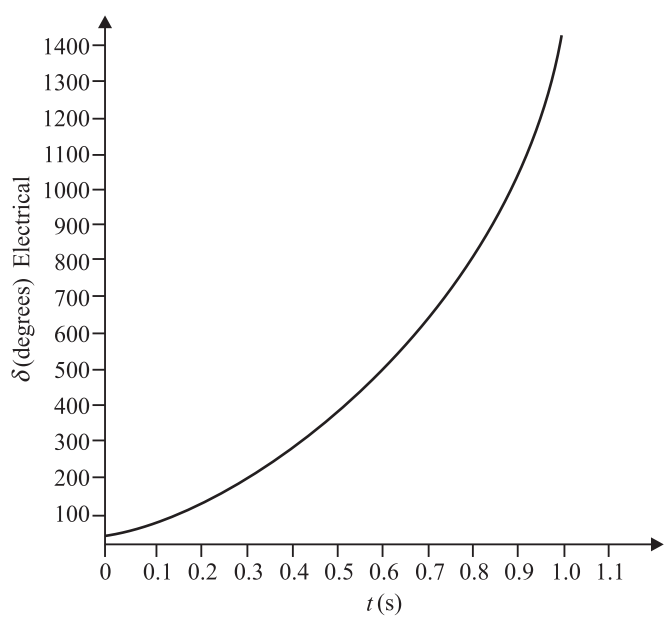
Time	*P*_max	sin(del)	*Pe*	*Pa*	9**Pa*	Del_ang	Delta
0–	2.333	0.42863	1	—	—	—	25.381
0+	0.875	0.42863	0.37505	0.62495	—	—	25.381
0avg	—	—	—	0.31247	2.8123	—	—
0.05	0.875	0.47244	0.41339	0.58661	5.2795	2.8123	28.193
0.1	0.875	0.5918	0.51782	0.48218	4.3396	8.0918	36.285
0.15	0.875	0.75145	0.65752	0.34248	3.0823	12.431	48.716
0.2	0.875	0.90055	0.78798	0.21202	1.9082	15.514	64.23
0.25	0.875	0.9894	0.86573	0.13427	1.2084	17.422	81.652
0.3	0.875	0.98394	0.86095	0.13905	1.2515	18.63	100.28
0.35	0.875	0.86459	0.75652	0.24348	2.1913	19.882	120.16
0.4–	0.875	0.6124	0.53585	0.46415	4.1774	22.073	142.24
0.4+	1.75	0.6124	1.0717	–0.0717	–0.6452	22.073	142.24
0.4avg	1.3125	0.6124	0.80377	0.19623	1.7661	22.073	142.24
0.45	1.75	0.24063	0.42111	0.57889	5.21	23.839	166.08
0.5	1.75	–0.26093	–0.4566	1.4566	13.11	29.049	195.13

(*Contd...*)

(*Contd...*)

Time	*P*_max	sin(del)	*Pe*	*Pa*	9**Pa*	Del_ang	Delta
0.55	1.75	–0.84136	–1.4724	2.4724	22.251	42.159	237.28
0.6	1.75	–0.85086	–1.489	2.489	22.401	64.41	301.69
0.65	1.75	0.47725	0.83519	0.16481	1.4833	86.811	388.51
0.7	1.75	0.89258	1.562	–0.5620	–5.0582	88.295	476.8
0.75	1.75	–0.34263	–0.5996	1.5996	14.396	83.237	560.04
0.8	1.75	–0.88564	–1.5499	2.5499	22.949	97.633	657.67
0.85	1.75	0.85037	1.4881	–0.4881	–4.3933	120.58	778.25
0.9	1.75	0.096883	0.16954	0.83046	7.4741	116.19	894.44
0.95	1.75	–0.8821	–1.5437	2.5437	22.893	123.66	1018.1
1	1.75	0.99566	1.7424	–0.7424	–6.6816	146.56	1164.7
1.05	1.75	–0.70131	–1.2273	2.2273	20.046	139.87	1304.5

critical clearing angle = 98.963
critical clearing time = 0.026711

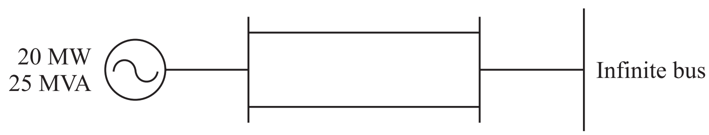


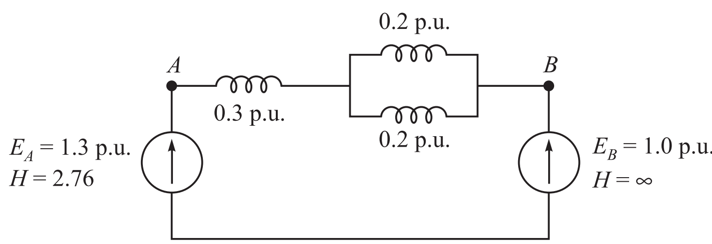
Initial displacement angle, δ_0 = 25.3808.

From the swing curve, the displacement angle δ goes on increasing. Hence the system is unstable.

EXAMPLE 5.17 A 25 MVA, 60 Hz, water wheel generator delivers 20 MW over a double circuit transmission line to a large metropolitan system which may be regarded as an infinite bus. The generating unit has a kinetic energy of 2.76 MJ/MVA at rated speed. The direct axis transient reactance of the generator is 0.3 p.u. The transmission circuits have negligible resistances and each has a reactance of 0.2 p.u. on a 25 MVA base. The voltage behind the transient reactance of the generator is 1.03 p.u. and the voltage of the metropolitan system is 1.0 p.u. A three-phase short circuit occurs at the middle of the transmission line circuit and is cleared in 0.4 s by the simultaneous opening of the circuit breaker at both ends of the line. Calculate and plot the swing curve of the generator for 1 s.

Solution:

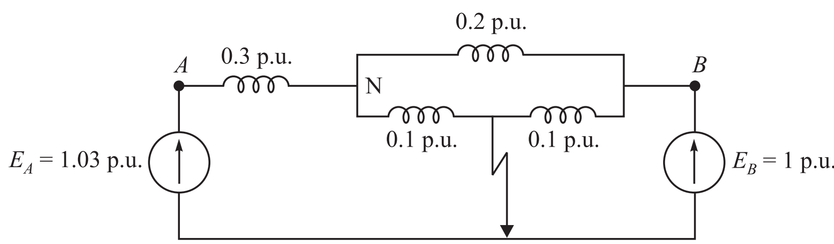




The net reactance for the prefault condition is given by

$$X_1 = X_{AB} = 0.3 + \frac{0.2 \times 0.2}{0.2 + 0.2} = 0.4 \text{ p.u.}$$

During the fault condition



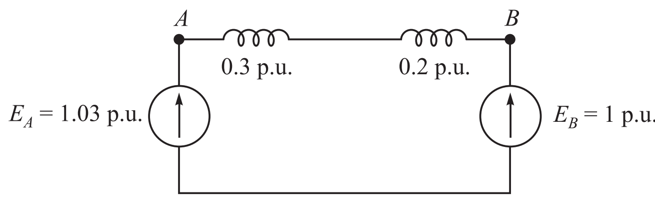
$$X_2 = X_{AB} = \frac{0.3 \times 0.2 + 0.2 \times 0.1 + 0.1 \times 0.3}{0.1}$$

$$= \frac{0.06 + 0.02 + 0.03}{0.1} = 1.1 \text{ p.u.}$$

$$X_2 = 1.1 \text{ p.u.}$$

During the postfault condition

$$X_3 = X_{AB} = 0.3 + 0.2 = 0.5 \text{ p.u.}$$



Prefault power, $P_{\max 1} = \frac{1.03 \times 1}{0.4} = 2.58$ p.u.

During fault power, $P_{\max 2} = \frac{1.03 \times 1}{1.1} = 0.936$ p.u.

Postfault power, $P_{\max 3} = \frac{1.03 \times 1}{0.5} = 2.06$ p.u.

$$M = \frac{SH}{180f} = \frac{1 \times 2.76}{180 \times 60} = 2.56 \times 10^{-4}$$

Output, $P_e = \frac{20}{25} = 0.8$ p.u.

In general $P_e = P_{\max} \sin\delta$

For the prefault condition,

$$P_e = P_{\max 1} \sin\delta$$
$$0.8 = 2.58 \sin\delta$$
$$\sin\delta = \frac{0.8}{2.58}$$
$$\delta = \sin^{-1}\frac{0.8}{2.58} = 18.2°$$

For the during fault condition, $P_e = P_{\max 2} \sin\delta$

For the postfault condition, $P_e = P_{\max 3} \sin\delta$

Immediately after the fault, δ remains unchanged momentarily but output changes.

$$P_e = P_{\max 2} \sin\delta = 0.936 \sin 18.2°$$
$$= 0.292 \text{ p.u.}$$

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{\Delta t^2}{M} P_{a(n-1)}$$
$$= \Delta\delta_{n-1} + \frac{(0.05)^2}{2.564 \times 10^{-4}} P_{a(n-1)}$$
$$= \Delta\delta_{n-1} + 9.76 P_{a\,(n-1)}$$

The calculation of δ versus t is done in the following table.

$$P_a = P_i - P_e = 0.8 - P_e$$

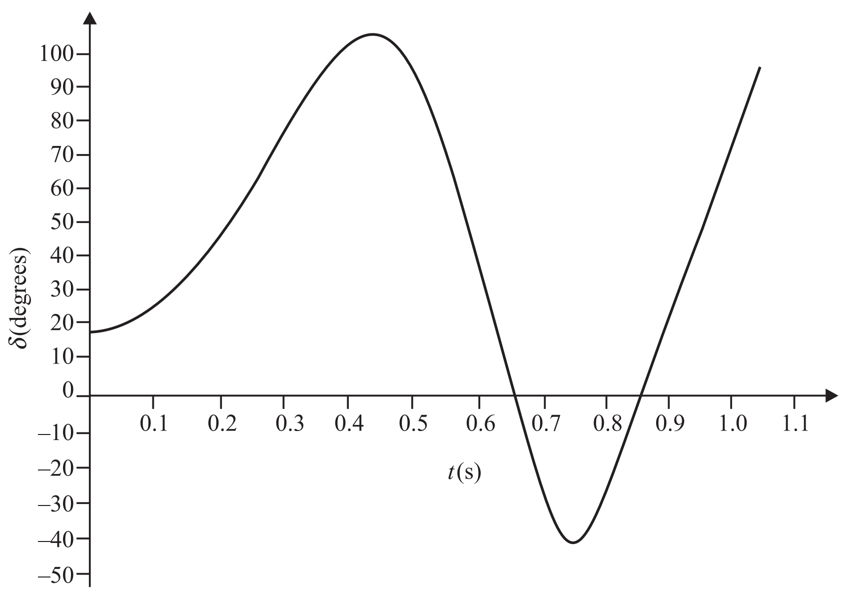
t(s)	P_{max}	sin δ	P_u	P_a	9.76P_a	$\Delta\delta$ degree	δ degree
0^-	2.58	0.31	0.8	0	—	—	18.1
0^+	0.936	0.31	0.29	0.51	—	—	18.1
0_{av}	—	—	—	0.255	2.5	—	—
						2.5	
0.05	0.936	0.352	0.33	0.47	4.6	—	20.6
						7.1	
0.1	0.936	0.465	0.435	0.365	3.562	—	27.7
						10.662	
0.15	0.936	0.621	0.581	0.219	2.138	—	38.362
						12.800	
0.2	0.936	0.779	0.729	0.071	0.692	—	51.162
						13.492	
0.25	0.936	0.904	0.846	–0.046	–0.448	—	64.654
						13.044	
0.3	0.936	0.977	0.915	–0.115	–1.118	—	77.698
						11.926	
0.35	0.936	1.0	0.936	–0.136	–1.327	—	89.624
						10.599	
0.4^-	0.936	0.984	0.921	–0.121	—	—	100.223
0.4^+	2.06	0.983	2.025	–1.125	—	—	100.223
0.4_{av}	—	—	—	–0.623	–6.568	—	—
						4.031	
0.45	2.06	0.969	1.997	–1.197	–11.679	—	104.254
						–7.648	
0.5	2.06	0.993	2.046	–1.246	–12.164	—	96.606
						–19.812	
0.55	2.06	0.974	2.006	–1.206	–11.766	—	76.794
						–31.578	
0.6	2.06	0.710	1.462	–0.662	–6.462	—	45.216
						–38.040	
0.65	2.06	0.125	0.257	0.543	5.297	—	7.176
						–32.743	
0.7	2.06	–0.432	–0.890	1.690	16.494	—	–25.567
						–16.249	
0.75	2.06	–0.667	–1.373	2.173	21.213	—	–41.816
						4.964	

(*Contd...*)

(*Contd*...)

t(s)	P_{max}	sin δ	P_u	P_a	9.76P_a	$\Delta\delta$ degree	δ degree
0.8	2.06	–0.600	–1.235	2.035	19.866	—	–36.852
						24.83	
0.85	2.06	–0.208	–0.429	1.229	11.996		–12.022
						36.826	
0.9	2.06	0.420	0.864	–0.064	–0.627	—	24.804
						36.199	
0.95	2.06	0.875	1.802	–1.002	–9.777	—	61.003
						26.422	
1.0	2.06	0.999	2.058	–1.258	–12.277	—	87.425

The variation of δ with time t is plotted as shown below.



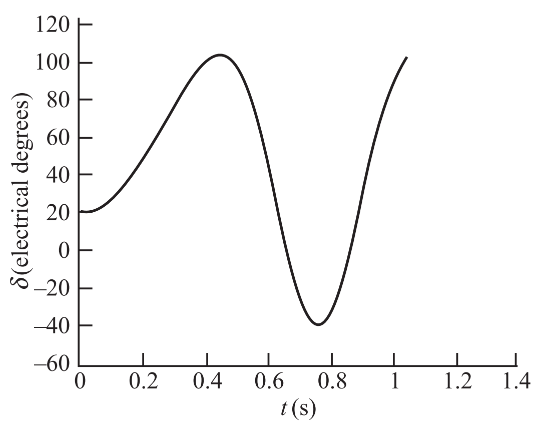
If we plot the swing curve, the value of δ increases and then decreases, this makes the system stable.

Results:

Enter the power limits for prefault, during fault, and postfault conditions: [2.58 .936 2.06]
Enter the frequency: 60
Enter the fault clearing time: 0.4
Enter the value of inertia constant: 2.76
Enter the initial displacement angle: 18.2
Enter the initial steady state power: 0.8.

t(s)	P_{max}	sin δ	Pe	Pa	9*Pa	$\Delta\delta$ degree	δ degree
0–	2.58	0.31233	0.8	0	—	0	18.2
0+	0.936	0.31233	0.29235	0.50765	—	0	18.2
0avg	—	—	—	0.255	2.5	0	18.2
0.05	0.936	0.33717	0.3156	0.4844	4.7387	1.5048	19.705
0.1	0.936	0.43756	0.40956	0.39044	3.8195	6.2436	25.948
0.15	0.936	0.58795	0.55032	0.24968	2.4425	10.063	36.012
0.2	0.936	0.74915	0.70121	0.098792	0.96644	12.506	48.517
0.25	0.936	0.88286	0.82636	–0.02636	–0.2578	13.472	61.989
0.3	0.936	0.96684	0.90496	–0.10496	–1.0268	13.214	75.204
0.35	0.936	0.99896	0.93503	–0.13503	–1.3209	12.187	87.391
0.4^-	0.936	0.98963	0.9263	–0.1263	–1.2355	10.867	98.257
0.4^+	2.06	0.98963	2.0386	–1.2386	–12.117	10.867	98.257
0.4avg	1.498	0.98963	1.4825	–0.68247	–6.6763	10.867	98.257
0.45	2.06	0.97649	2.0116	–1.2116	–11.852	4.1902	102.45
0.5	2.06	0.99651	2.0528	–1.2528	–12.256	–7.6622	94.785
0.55	2.06	0.96532	1.9886	–1.1886	–11.627	–19.918	74.867
0.6	2.06	0.6861	1.4134	–0.61336	–6.0003	–31.545	43.322
0.65	2.06	0.10065	0.20733	0.59267	5.7978	–37.546	5.7764
0.7	2.06	–0.4379	–0.9021	1.7021	16.651	–31.748	–25.971
0.75	2.06	–0.6569	–1.3533	2.1533	21.065	–15.097	–41.068
0.8	2.06	–0.575	–1.1845	1.9845	19.414	5.9685	–35.099
0.85	2.06	–0.1687	–0.3477	1.1477	11.228	25.382	–9.7174
0.9	2.06	0.45231	0.93176	–0.13176	–1.289	36.61	26.892
0.95	2.06	0.88468	1.8225	–1.0225	–10.002	35.321	62.213
1	2.06	0.99907	2.0581	–1.2581	–12.307	25.318	87.531
1.05	2.06	0.98312	2.0252	–1.2252	–11.986	13.011	100.54

critical clearing angle = 137.85
critical clearing time = 0.036526



Swing curve

In the swing curve, the value of δ increases and then decreases, this makes the system stable.

5.8.3 Euler's Method

The Euler's method is the simplest and the least accurate of all numerical methods. It is presented here because of its simplicity. By studying this method, we will be able to grasp the basic ideas involved in the numerical solutions of one-dimensional equation (ODE) and can easily understand the comparatively complex method such as the Runge–Kutta procedure.

Let us consider the first order differential equation

$$\frac{dx}{dt} = f(x, t) \tag{5.41}$$

Figure 5.29 illustrate the principles of applying the Euler's method at initial condition $x = x_0$ at $t = t_0$.

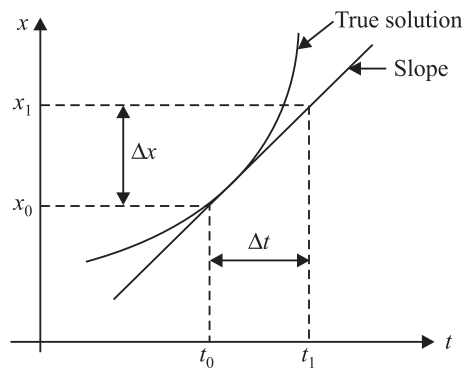


Figure 5.29 Graphical interpretation of Euler's method.

At $x = x_0$, $t = t_0$, we can approximate the curve representing the true solution by its tangent having a slope

$$\left.\frac{dx}{dt}\right|_{x=x_0} = f(x_0, t_0) \tag{5.42}$$

For a small increment in t denoted by Δt, the increment in x is given by Δx Therefore,

$$\Delta x = \left.\frac{dx}{dt}\right|_{x=x_0} \cdot \Delta t \tag{5.43}$$

where $\left.\frac{dx}{dt}\right|_{x=x_0}$ is the slope of the curve at (t_0, x_0), which can be determined from Eq. (5.42).Thus, the value of x at $t = t_1 = t_0 + \Delta t$ is given by

$$x_1 = x_0 + \Delta x = x_0 + \left.\frac{dx}{dt}\right|_{x=x_0} \cdot \Delta t \tag{5.44}$$

The Euler's method is equivalent to using the first two terms of the Taylor series expansion for x around the point (t_0, x_0)

$$x_1 = x_0 + \Delta t(\dot{x}_0) + \frac{\Delta t^2}{2!}(\ddot{x}_0) + \frac{\Delta t^3}{3!}(\dddot{x}_0) + \cdots \tag{5.45}$$

After using the Euler's technique for determining $x = x_1$ corresponding to $t = t_1$, we can take another short time step Δt and determine x_2 corresponding to $t_2 = t_1 + \Delta t$ as follows:

$$x_2 = x_1 + \left.\frac{dx}{dt}\right|_{x=x_1} \cdot \Delta t \tag{5.46}$$

The subsequent values of x can be similarly determined. Hence, the computational algorithm is

$$x_{i+1} = x_i + \left.\frac{dx}{dt}\right|_{x=x_i} \cdot \Delta t \tag{5.47}$$

By applying the above algorithm successively, the values of x can be determined corresponding to different values of t.

The method considers only the first derivative of x and is, therefore, referred to as a first order method. For sufficient accuracy at each step Δt has to be small. This will increase round-off errors, and the computational effort required will be very high.

5.8.4 Modified Euler's Method

The standard Euler's method results in inaccuracies because it uses the derivative at the beginning of the interval though it applied throughout the interval. The slope is constant over the entire interval Δt causing the points to fall below the curve. The above problem is solved by the modified Euler's method. This method can be obtained by calculating the slope both at the beginning and the end of the interval, and then averaging these slopes. This procedure is known as the modified Euler's method.

The modified Euler's method consists of the following steps.

(i) **Predictor step:** By using the derivative at the beginning of the step, the value at the end of the step ($t_1 = t_0 + \Delta t$) is predicted as

$$x_1^p = x_0 + \left.\frac{dx}{dt}\right|_{x=x_0} \cdot \Delta t \tag{5.48}$$

(ii) **Corrector step:** By using the predicted value of x_1^p, the derivative at the end of the step is computed and the value at the end of the step($t_1 = t_0 + \Delta t$) is predicted as

$$\left.\frac{dx}{dt}\right|_{x=x_1^p} = f(x_1^p, t_1) \tag{5.49}$$

Then, the average value of the two derivatives is used to find the corrected value.

$$x_1^c = x_0 + \left(\frac{\left.\frac{dx}{dt}\right|_{x=x_0} + \left.\frac{dx}{dt}\right|_{x=x_1^p}}{2} \right) \cdot \Delta t \tag{5.50}$$

Similarly,

$$x_2^c = x_1 + \left(\frac{\left.\frac{dx}{dt}\right|_{x=x_1} + \left.\frac{dx}{dt}\right|_{x=x_2^p}}{2} \right) \cdot \Delta t \tag{5.51}$$

$$x_{i+}^c = x_i + \left(\frac{\left.\frac{dx}{dt}\right|_{x=x_i} + \left.\frac{dx}{dt}\right|_{x=x_{i+1}^p}}{2} \right) \cdot \Delta t \tag{5.52}$$

This process can be repeated until the successive steps converge with the desired accuracy.

5.8.5 Runge–Kutta (R–K) Method

The Runge–Kutta method approximates the Taylor series solution; however, unlike the formal Taylor series solution, the R–K method does not require explicit evaluation of derivatives higher than the first. The effects of the higher derivatives are included by several evaluations of the first derivative depending on the number of terms effectively retained in the Taylor series, we have R–K method of different orders.

Let us consider the first order differential equation

$$\frac{dx}{dt} = f(x, t) \tag{5.53}$$

Assume the initial condition is x_0, t_0.

Second order R–K method

The second order R–K formula for the value of x at $t = t_0 + \Delta t$ is given by

$$x_1 = x_0 + \Delta x = x_0 + \frac{k_1 + k_2}{2} \tag{5.54}$$

where

k_1 = (slope at the beginning of time step) $\Delta t = f(x_0, t_0)\Delta t$

k_2 = (first approximation to slope at midstep) $\Delta t = f(x_0 + k_1, t_0 + \Delta t)\Delta t$

Fourth order R-K method

The general formula giving the value of x for the $(n + 1)$th step

$$x_{i+1} = x_i + \Delta x = x_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{5.55}$$

where

k_1 = (slope at the beginning of time step) $\Delta t = f(x_i, t_i)\Delta t$

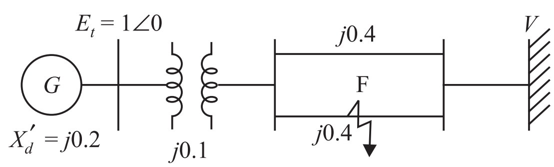
k_2 = (first approximation to slope at midstep) $\Delta t = f\left(x_i + \frac{k_1}{2}, t_i + \frac{\Delta t}{2}\right)\Delta t$

k_3 = (second approximation to slope at midstep) $\Delta t = f\left(x_i + \frac{k_2}{2}, t_i + \frac{\Delta t}{2}\right)\Delta t$

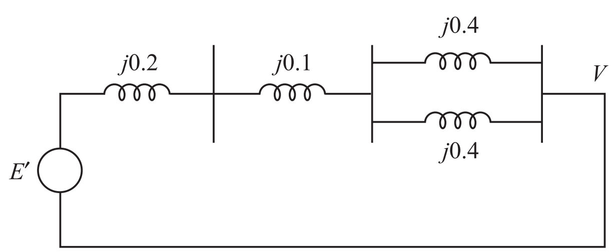
k_4 = (slope at the end of step) $\Delta t = f(x_i + k_3, t_i + \Delta t)\Delta t$

Thus Δx is the incremental value of x given by the weighted average of estimated based on slopes at the beginning of the mid-point and end of the time step.

EXAMPLE 5.18 A three-phase fault occurs at the point F as shown in the figure is cleared by isolating the faulted circuit simultaneously from both ends. Generator is delivering 0.8 p.u. power at 0.8 p.f lagging. The fault is cleared in 0.1 s. Obtain the numeric solution of the swing equation up to 0.15 s using the (a) modified Euler's method and (b) Runge–Kutta method with step size of Δt = 0.05 s. Take H = 5 MJ/MVA.



Solution:



(i) To draw the reactance diagram

$E_t = 1.0\Delta\angle 0°$

$P = 0.8$ p.u.; $\cos\theta = 0.8$; $\theta = \cos^{-1}(0.8) = 36.86° = 0.644$ rad

$Q = P\tan\theta = 0.6$

$$I = \frac{P - jQ}{E_t} = \frac{0.8 - j0.6}{1.0} = 0.8 - j0.6$$

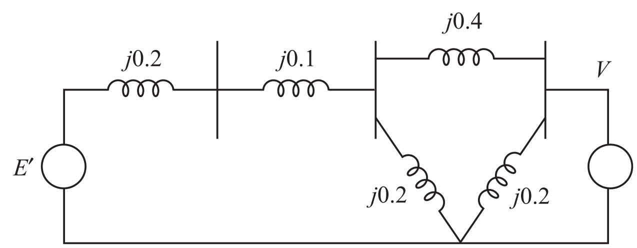
$$E' = E_t + jX'_d I = 1.0 + j0.2(0.8 - j0.6) = 1.13\angle 0.142°$$

$$V = E_t - j(X_{\text{trans}} + X_{t.\text{line}})I = 1.13 + j(0.1 + 0.2)\ (0.8 - j0.6)$$
$$= 0.854\angle{-0.285°}$$

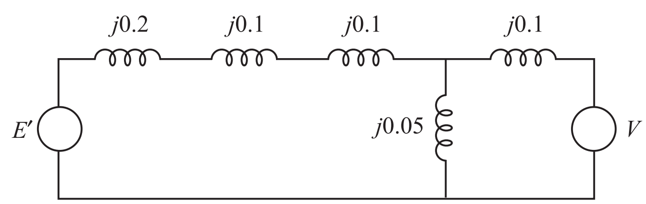
(ii) *Prefault condition* From the reactance diagram,

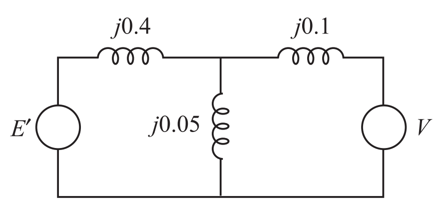
$$X_T = X_I = 0.2 + 0.1 + \left(\frac{0.4 \times 0.4}{0.4 + 0.4}\right) = 0.5$$

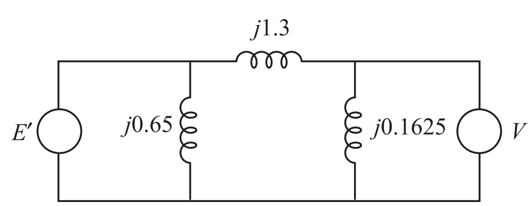
$$P_{e1} = \frac{|E'||V|}{X_t}\sin\delta = \frac{1.13 \times 0.854}{0.5}\sin\delta = 1.93\sin\delta$$



(iii) *During the fault condition* (fault at the middle of the transmission line 2)
Using delta to star conversion, the circuit becomes



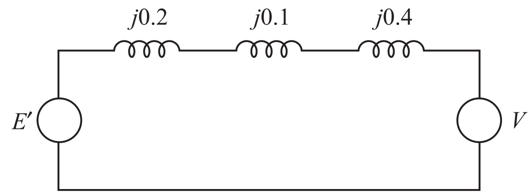




From the reactance diagram, $X_T = X_{II} = 1.3$

$$P_{e2} = \frac{|E'||V|}{X_{II}}\sin\delta = \frac{1.13 \times 0.854}{1.3}\sin\delta = 0.742\sin\delta$$

(iv) *Postfault condition*



From the reactance diagram, $X_T = X_{III} = j0.7$

$$P_{e3} = \frac{|E'||V|}{X_{III}} \sin\delta = \frac{1.13 \times 0.854}{0.7} \sin\delta = 1.378 \sin\delta$$

(a) Modified Euler's method

During fault,

$$P_e = 0.742 \sin\delta$$
$$P_{e0} = P_{m0} = 0.8 = 1.93 \sin\delta_0$$
$$\delta_0 = \sin^{-1}\left(\frac{0.8}{1.93}\right) = 24.48° = 0.427 \text{ rad}$$
$$\omega_0 = 2\pi f = 2\pi \times 50 = 314.159$$
$$\Delta t = 0.05 \text{ s}$$

Iteration 1: Beginning of the first step at, $t = 0$

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_0} = \Delta\omega_0 = \omega_0 - 2\pi f = 314.159 - 314.159 = 0$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_0} = \frac{\pi f}{H}(P_m - P_e(\delta_0)) = \frac{\pi \times 50}{5}(0.8 - 0.742 \sin 0.427) = 15.479$$

End of the first step, $t = 0.05$
Predicted values are

$$\delta^p_{0.05} = \delta_0 + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_0} \times \Delta t = 0.427 + 0 \times 0.05 = 0.427 \text{ rad}$$

$$\Delta\omega^p_{0.05} = \Delta\omega_0 + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_0} \times \Delta t = 0 + 15.479 \times 0.05 = 0.774 \text{ rad/s}$$

Derivatives at the end of $t = 0.05$

$$t = 0.05, \quad \left.\frac{d\delta}{dt}\right|_{\Delta\omega^p_{0.05}} = \Delta\omega^p_{0.05} = 0.774 \text{ rad/s}$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta^p_{0.05}} = \frac{\pi f}{H}(P_m - P_e(\delta^p_{0.05})) = \frac{\pi \times 50}{5}(0.8 - 0.742 \sin 0.427) = 15.479 \text{ rad/s}$$

The corrected values are

$$\delta_{0.05}^{c} = \delta_0 + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_0} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.05}^{p}}}{2}\right] \times \Delta t = 0.427 + \left[\frac{0+0.774}{2}\right] \times 0.05 = 0.446\,\text{rad}$$

$$\Delta\omega_{0.05}^{c} = \Delta\omega_0 + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_0} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.05}^{p}}}{2}\right] \times \Delta t = 0 + \left[\frac{15.479+15.479}{2}\right] \times 0.05$$

$$= 0.774 \text{ rad/s}$$

Iteration 2: Beginning of the first step at t = 0.05,

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.05}^{c}} = \Delta\omega_{0.05}^{c} = 0.774$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.05}^{c}} = \frac{\pi f}{H}(P_m - P_e(\delta_{0.05}^{c})) = \frac{\pi\times 50}{5}(0.8 - 0.742\sin 0.446) = 15.077$$

End of the second step, t = 0.1
Predicted values are

$$\delta_{0.1}^{p} = \delta_{0.05}^{c} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.05}^{c}} \times \Delta t = 0.446 + 0.774\times 0.05 = 0.485 \text{ rad}$$

$$\Delta\omega_{0.1}^{p} = \Delta\omega_{0.05}^{c} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.05}^{c}} \times \Delta t = 0.774 + 15.077\times 0.05 = 1.5278 \text{ rad/s}$$

Derivatives at the end of t = 0.1,

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^{p}} = \Delta\omega_{0.1}^{p} = 1.5278 \text{ rad/s}$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}^{p}} = \frac{\pi f}{H}(P_m - P_e(\delta_{0.1}^{p})) = \frac{\pi\times 50}{5}(0.8 - 0.742\sin 0.485) = 14.265 \text{ rad/s}$$

The corrected values are

$$\delta_{0.1}^{c} = \delta_{0.05}^{c} + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.05}^{c}} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^{p}}}{2}\right] \times \Delta t = 0.446 + \left[\frac{0.774+1.5278}{2}\right] \times 0.05$$

$$= 0.5035 \text{ rad}$$

$$\Delta\omega_{0.1}^{c} = \Delta\omega_{0.05}^{c} + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.05}^{c}} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}^{p}}}{2}\right] \times \Delta t$$

$$= 0.774 + \left[\frac{15.077+14.265}{2}\right] \times 0.05 = 1.5076 \text{ rad/s}$$

The fault is cleared at $t = 0.1$ s and the postfault condition is given by

$$P_e = 1.378 \sin\delta$$

Iteration 3: Beginning of the first step at 0.1 s

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^c} = \Delta\omega_{0.1}^c = 1.5076$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}^c} = \frac{\pi f}{H}(P_m - P_e(\delta_{0.1}^c)) = \frac{\pi\times 50}{5}(0.8 - 1.378\sin 0.5035) = 4.245$$

End of the third step, $t = 0.15$ s
Predicted values are

$$\delta_{0.15}^p = \delta_{0.1}^c + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^c} \times \Delta t = 0.5035 + 1.5076\times 0.05 = 0.579 \text{ rad}$$

$$\Delta\omega_{0.15}^p = \Delta\omega_{0.1}^c + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}^c} \times \Delta t = 1.5076 + 4.245\times 0.05 = 1.7197 \text{ rad/s}$$

Derivatives at the end of $t = 0.15$ s,

$$\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.15}^p} = \Delta\omega_{0.15}^p = 1.7197 \text{ rad/s}$$

$$\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.15}^p} = \frac{\pi f}{H}(P_m - P_e(\delta_{0.15}^p)) = \frac{\pi\times 50}{5}(0.8 - 1.378\sin 0.579) = 1.444 \text{ rad/s}$$

The corrected values are

$$\delta_{0.15}^c = \delta_{0.1}^c + \left[\frac{\left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.1}^c} + \left.\frac{d\delta}{dt}\right|_{\Delta\omega_{0.15}^p}}{2}\right]\times\Delta t$$

$$= 0.5035 + \left[\frac{1.5076 + 1.7197}{2}\right]\times 0.05 = 0.584 \text{ rad}$$

$$\Delta\omega_{0.15}^c = \Delta\omega_{0.1}^c + \left[\frac{\left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.1}^c} + \left.\frac{d\Delta\omega}{dt}\right|_{\delta_{0.15}^p}}{2}\right]\times\Delta t$$

$$= 1.5076 + \left[\frac{4.245 + 1.444}{2}\right]\times 0.05 = 1.6498 \text{ rad/s}$$

(b) Runge–Kutta method

$$P_e = 0.742 \sin\delta$$

$$P_{e0} = P_{m0} = 0.8 = 1.93 \sin\delta_0$$

$$\delta_0 = \sin^{-1}\left(\frac{0.8}{1.93}\right) = 24.48° = 0.427 \text{ rad}$$

$$\omega_0 = 2\pi f = 2\pi \times 50 = 314.159$$

$$\Delta t = 0.05 \text{ s}$$

Fourth order method

Iteration 1: at $t = 0$,

Ist estimate: $k_1 = \Delta\omega_0 \times \Delta t = 0 \times 0.05 = 0$

$$l_1 = \frac{\pi f}{H}(P_m - P_e(\delta_0))\Delta t = \frac{\pi \times 50}{5}(0.8 - 0.742 \sin 0.427) \times 0.05 = 0.774$$

IInd estimate: $k_2 = \left(\Delta\omega_0 + \frac{l_1}{2}\right) \times \Delta t = \left(0 + \frac{0.774}{2}\right) \times 0.05 = 0.0194$

$$l_2 = \frac{\pi f}{H}\left(P_m - P_e\left(\delta_0 + \frac{k_1}{2}\right)\right)\Delta t = \frac{\pi \times 50}{5}\left(0.8 - 0.742 \sin\left(0.427 + \frac{0}{2}\right)\right) \times 0.05$$

$$= 0.774$$

IIIrd estimate: $k_3 = \left(\Delta\omega_0 + \frac{l_2}{2}\right) \times \Delta t = \left(0 + \frac{0.774}{2}\right) \times 0.05 = 0.0194$

$$l_3 = \frac{\pi f}{H}\left(P_m - P_e\left(\delta_0 + \frac{k_2}{2}\right)\right)\Delta t = \frac{\pi \times 50}{5}\left(0.8 - 0.742 \sin\left(0.427 + \frac{0.0194}{2}\right)\right) \times 0.05$$

$$= 0.764$$

IVth estimate: $k_4 = (\Delta\omega_0 + l_3) \times \Delta t = (0 + 0.764) \times 0.05 = 0.038$

$$l_4 = \frac{\pi f}{H}(P_m - P_e(\delta_0 + k_3))\Delta t = \frac{\pi \times 50}{5}(0.8 - 0.742 \sin(0.427 + 0.0194)) \times 0.05$$

$$= 0.7535$$

Final estimate:

$$\delta_{0.05} = \delta_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 0.427 + \frac{1}{6}(0 + 2 \times 0.0194 + 2 \times 0.0194 + 0.038)$$

$$= 0.446 \text{ rad}$$

$$\Delta\omega_{0.05} = \Delta\omega_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

$$= 0 + \frac{1}{6}(0.774 + 2 \times 0.774 + 2 \times 0.764 + 0.7535)$$

$$= 0.767 \text{ rad/s}$$

Iteration 2: at $t = 0.05$s,

Ist estimate: $k_1 = \Delta\omega_{0.05} \times \Delta t = 0.767 \times 0.05 = 0.0384$

$$l_1 = \frac{\pi f}{H}(P_m - P_e(\delta_{0.05}))\Delta t = \frac{\pi \times 50}{5}(0.8 - 0.742 \sin 0.446) \times 0.05 = 0.754$$

IInd estimate: $k_2 = \left(\Delta\omega_{0.05} + \frac{l_1}{2}\right) \times \Delta t = \left(0.767 + \frac{0.754}{2}\right) \times 0.05 = 0.0572$

$$l_2 = \frac{\pi f}{H}\left(P_m - P_e\left(\delta_{0.05} + \frac{k_1}{2}\right)\right)\Delta t$$

$$= \frac{\pi \times 50}{5}\left(0.8 - 0.742 \sin\left(0.446 + \frac{0.0384}{2}\right)\right) \times 0.05$$

$$= 0.734$$

IIIrd estimate: $k_3 = \left(\Delta\omega_{0.05} + \frac{l_2}{2}\right) \times \Delta t = \left(0.767 + \frac{0.734}{2}\right) \times 0.05 = 0.0567$

$$l_3 = \frac{\pi f}{H}\left(P_m - P_e\left(\delta_{0.05} + \frac{k_2}{2}\right)\right)\Delta t$$

$$= \frac{\pi \times 50}{5}\left(0.8 - 0.742 \sin\left(0.446 + \frac{0.0572}{2}\right)\right) \times 0.05$$

$$= 0.724$$

IVth estimate: $k_4 = (\Delta\omega_{0.05} + l_3) \times \Delta t = (0.767 + 0.724) \times 0.05 = 0.0746$

$$l_4 = \frac{\pi f}{H}(P_m - P_e(\delta_{0.05} + k_3))\Delta t$$

$$= \frac{\pi \times 50}{5}(0.8 - 0.742 \sin(0.446 + 0.0567)) \times 0.05$$

$$= 0.695$$

Final estimate:

$$\delta_{0.1} = \delta_{0.05} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 0.446 + \frac{1}{6}(0.0384 + 2 \times 0.0572 + 2 \times 0.0567 + 0.0746)$$

$$= 0.503 \text{ rad}$$

$$\Delta\omega_{0.1} = \Delta\omega_{0.05} + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

$$= 0.767 + \frac{1}{6}(0.754 + 2 \times 0.734 + 2 \times 0.724 + 0.695)$$

$$= 1.495 \text{ rad/s}$$

Fault is cleared at $t = 0.1$

Postfault condition, $P_e = 1.378 \sin\delta$

Iteration 3:

Ist estimate: $k_1 = \Delta\omega_{0.1} \times \Delta t = 1.495 \times 0.05 = 0.0748$

$$l_1 = \frac{\pi f}{H}(P_m - P_e(\delta_{0.1}))\,\Delta t = \frac{\pi \times 50}{5}(0.8 - 1.378 \sin 0.503) \times 0.05 = 0.213$$

IInd estimate: $k_2 = \left(\Delta\omega_{0.1} + \frac{l_1}{2}\right) \times \Delta t = \left(1.495 + \frac{0.213}{2}\right) \times 0.05 = 0.08$

$$l_2 = \frac{\pi f}{H}\left(P_m - P_e\left(d_{0.1} + \frac{k_1}{2}\right)\right)\Delta t$$

$$= \frac{\pi \times 50}{5}\left(0.8 - 1.378 \sin\left(0.503 + \frac{0.748}{2}\right)\right) \times 0.05$$

$$= 0.143$$

IIIrd estimate: $k_3 = \left(\Delta\omega_{0.1} + \frac{l_2}{2}\right) \times \Delta t = \left(1.495 + \frac{0.143}{2}\right) \times 0.05 = 0.0783$

$$l_3 = \frac{\pi f}{H}\left(P_m - P_e\left(\delta_{0.1} + \frac{k_2}{2}\right)\right)\Delta t$$

$$= \frac{\pi \times 50}{5}\left(0.8 - 1.378\sin\left(0.503 + \frac{0.08}{2}\right)\right) \times 0.05$$

$$= 0.1382$$

IVth estimate: $k_4 = (\Delta\omega_{0.1} + l_3) \times \Delta t = (1.495 + 0.1382) \times 0.05 = 0.0817$

$$l_4 = \frac{\pi f}{H}(P_m - P_e(\delta_{0.1} + k_3))\Delta t$$

$$= \frac{\pi \times 50}{5}(0.8 - 1.378 \sin(0.503 + 0.0783)) \times 0.05$$

$$= 0.068$$

Final estimate:

$$\delta_{0.15} = \delta_{0.1} + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 0.503 + \frac{1}{6}(0.0748 + 2 \times 0.08 + 2 \times 0.0783 + 0.0817)$$

$$= 0.582 \text{ rad}$$

$$\Delta\omega_{0.15} = \Delta\omega_{0.1} + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

$$= 1.495 + \frac{1}{6}(0.213 + 2 \times 0.143 + 2 \times 0.1382 + 0.068)$$

$$= 1.636 \text{ rad/s}$$

5.9 Multimachine Transient Stability

Transient stability analysis has recently become a major issue in the operation of power system due to the increasing stress on power system networks. This problem requires evaluation of a power system's ability to withstand disturbances while maintaining the quality of service. Many different techniques have been proposed for transient stability analysis in power system, especially for a multi-machine system.

Multi-machine equations can be written similar to the one-machine system connected to the infinite bus. In order to reduce the complexity of the transient stability analysis, similar simplifying assumptions are made as follows.

- Each synchronous machine is represented by a constant voltage source behind the direct axis transient reactance. This representation neglects the effect of saliency and assumes constant flux linkages.
- The actions of the governor are neglected and the input powers are assumed to remain constant during the entire period of simulation.
- Using the prefault bus voltages, all loads are converted to equivalent admittances to ground and are assumed to remain constant.
- Damping or asynchronous powers are ignored.
- The mechanical rotor angle of each machine coincides with the angle of the voltage behind the machine reactance.
- Machines belonging to the same station swing together and are said to be coherent. A group of coherent machines is represented by one equivalent machine.

5.9.1 Mathematical Model of Multimachine Transient Stability Analysis

The first step in the transient stability analysis is to solve the initial load flow and to determine the initial bus voltage magnitudes and phase angles. The machine currents prior to disturbance are calculated from,

$$I_i = \frac{S_i^*}{V_i^*} = \frac{P_i - jQ_i}{V_i^*} \qquad i = 1, 2, ..., m \tag{5.56}$$

where m is the number of generators

V_i is the terminal voltage of the ith generator

P_i and Q_i are the generators of real and reactive powers.

All unknown values are determined from the initial power flow solution. The generator armature resistances are usually neglected and the voltages behind the transient reactance are then obtained as

$$E_i' = V_i + jX_d' I_i \tag{5.57}$$

Next, all loads are converted to equivalent admittances by using the relation

$$y_{i0} = \frac{S_i^*}{|V_i|^2} = \frac{P_i - jQ_i}{|V_i|^2} \tag{5.58}$$

To include voltages behind the transient reactance, m buses are added to the n bus power system network. The equivalent network, with all loads converted to admittances is shown in Figure 5.30.

Nodes $n + 1$, $n + 2$, ..., $n + m$ are the internal machine buses, i.e. the buses behind the transient reactances. The node voltage equation, with node 0 as reference for this network, is

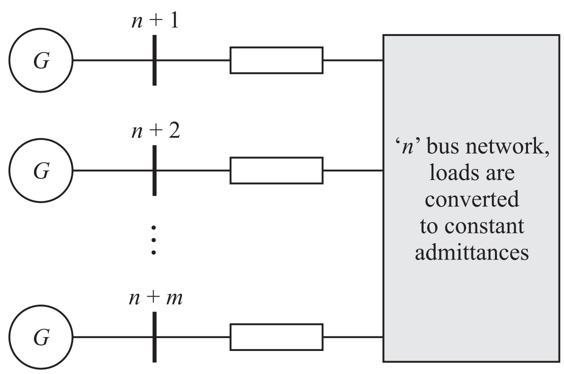


Figure 5.30 Power system representation for transient stability analysis.

$$\begin{bmatrix} I_1 \\ \vdots \\ I_n \\ I_{n+1} \\ \vdots \\ I_{n+m} \end{bmatrix} = \begin{bmatrix} Y_{11} & \cdots & Y_{n1} & Y_{1(n+1)} & \cdots & Y_{1(n+m)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ Y_{1n} & \cdots & Y_{nm} & Y_{n(n+1)} & \cdots & Y_{n(n+m)} \\ Y_{(n+1)1} & \cdots & Y_{(n+1)n} & Y_{(n+1)(n+1)} & \cdots & Y_{(n+1)(n+m)} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ Y_{(n+m)1} & \cdots & Y_{(n+m)n} & Y_{(n+m)(n+1)} & \cdots & Y_{(n+m)(n+m)} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_n \\ E'_{n+1} \\ \vdots \\ E'_{n+m} \end{bmatrix} \tag{5.59}$$

or

$$I_{\text{bus}} = Y_{\text{bus}} V_{\text{bus}} \tag{5.60}$$

where I_{bus} is the vector of the injected bus currents

V_{bus} is the vector of bus voltages measured from the reference node.

The diagonal elements of the bus admittance matrix are the sum of admittances connected to it, and the off-diagonal elements are equal to the negative of the admittance between the nodes. The reference is that additional nodes are added to include the machine voltages behind transient reactances. Also, the diagonal elements are modified to include the load admittances.

To simplify the analysis, all nodes other than the generator internal nodes are eliminated using Kron's reduction formula. To eliminate the load buses, the bus admittance matrix in Eq. (5.59) is partitioned such that the n buses to be removed are represented in the upper n rows. Since no current enters or leaves the load buses, currents in the n rows is zero. The generator current is denoted by the vector I_m and the generator and load voltages are represented by the vectors E'_m and V_n, respectively. Then, Eq. (5.59), in terms of sub matrices becomes

$$\begin{bmatrix} 0 \\ I_m \end{bmatrix} = \begin{bmatrix} Y_{nn} & Y_{nm} \\ Y_{nm}^t & Y_{mm} \end{bmatrix} \begin{bmatrix} V_n \\ E'_m \end{bmatrix} \tag{5.61}$$

The voltage vector V_n may be eliminated upon substitution as follows.

$$0 = Y_{nn}V_n + Y_{nm}E'_m \tag{5.62}$$

$$I_m = Y_{nm}^t V_n + Y_{mm}E'_m \tag{5.63}$$

From Eq. (5.62)

$$V_n = -Y_{nn}^{-1} + Y_{nm}E'_m \tag{5.64}$$

Now substituting into Eq. (5.63)

$$I_m = [Y_{mm} - Y_{nm}^t Y_{nn}^{-1} Y_{nm}]\, E'_m = Y_{\text{bus}}^{\text{red}} E'_m \tag{5.65}$$

The reduced admittance matrix is

$$Y_{\text{bus}}^{\text{red}} = Y_{mm} - Y_{nm}^t Y_{nn}^{-1} Y_{nm} \tag{5.66}$$

The reduced bus admittance matrix has the dimensions ($m \times m$), where m is the number of generators. The electrical power output of each machine can now be expressed in terms of the machine's internal voltages

$$S_{ei}^* = E_i'^* I_i$$

or

$$P_{ei} = \text{Re}(E_i'^* I_i) \tag{5.67}$$

where

$$I_i = \sum_{j=1}^{m} E'_j Y_{ij} \tag{5.68}$$

Expressing voltages and admittances in the polar form,

$$E'_i = |E'_i| \angle \delta_i \quad \text{and} \quad Y_{ij} = |Y_{ij}| \angle \theta_{ij}$$

Substituting I_i in Eq. (5.67), we get

$$P_{ei} = \sum_{j=1}^{m} |E'_i|\,|E'_j|\,|Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \tag{5.69}$$

The above equation is the same as the power flow equation. Prior to disturbance, there is equilibrium between the mechanical power input and the electrical power output, and we have

$$P_{mi} = \sum_{j=1}^{m} |E_i'||E_j'||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \qquad (5.70)$$

The classical transient stability study is based on the application of a three-phase fault. A solid three-phase fault at bus k in the network results in $V_k = 0$. This is simulated by removing the kth row and column from the prefault bus admittance matrix. The new bus admittance matrix is reduced by eliminating all nodes except the internal generator nodes. The generator excitation voltages during the fault and the postfault modes are assumed to remain constant. The electrical power of the ith generator in terms of the new reduced bus admittance matrices are obtained from Eq. (5.69). The swing equation with damping neglected, for machine i becomes

$$\frac{H_i}{\pi f_0}\frac{d^2\delta_i}{dt^2} = P_{mi} - \sum_{j=1}^{m} |E_i'||E_j'||Y_{ij}|\cos(\theta_{ij} - \delta_i + \delta_j) \qquad (5.71)$$

where, Y_{ij} are the elements of the faulted reduced bus admittance matrix, H_i is the inertia constant of machine i expressed on the common MVA base SB. If H_{Gi} is the inertia constant of machine i expressed on the machine rated MVA S_{Gi}, then H_i is given by

$$H_i = \frac{S_{Gi}}{S_{cb}} H_{Gi} \qquad (5.72)$$

Showing the electrical power of the ith generator by P_{ef} and transforming Eq. (5.71) into state variable mode yield

$$\frac{d\delta_i}{dt} = \Delta\omega_i \qquad (5.73)$$

$$\frac{d\Delta\omega_i}{dt} = \frac{\pi f_0}{H_i}(P_{mi} - P_{ei}^{f}) \qquad (5.74)$$

In the transient stability analysis problem, we have two state equations for each generator. When the fault is cleared, which may involve the removal of the faulty line, the bus admittance matrix is recomputed to reflect the change in the network. Next the postfault reduced bus admittance matrix is evaluated and the postfault electrical power of the ith generator shown by P_i^{pf} readily determined. Using the postfault power P_i^{pf}, the simulation is continued to determine the system stability, until the plots reveal a definite trend as to stability or instability. Usually the slack generator is selected as the reference machines are plotted. Usually, the solution is carried out for two swings to show that the second swing is not greater than the first one. If the angle differences do not increase, the system is stable. If any of the angle differences increase indefinitely, the system is unstable. The flow chart of transient stability analysis for a multimachine power is given in Figure 5.31.

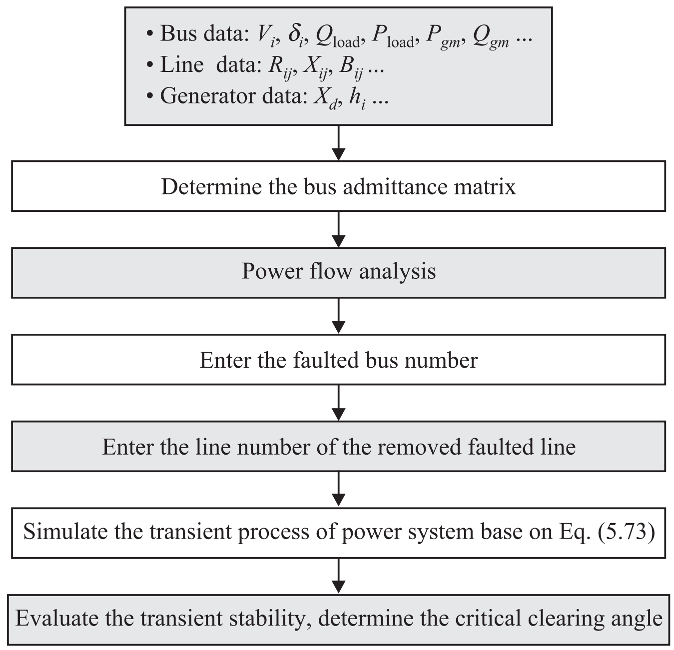


Figure 5.31 Flow chart of transient stability analysis for a multimachine power.

5.10 Factors Influencing Transient Stability

The factors that affect the transient stability are given below.

(i) *The generator inertia.* The higher the inertia, the lower is the rate of change in angle. This reduces the kinetic energy gained during fault.
(ii) *The generator reactance.* A lower reactance increases peak power and reduces initial rotor angle.
(iii) The generator internal voltage magnitude (E'). This depends on the field excitation.
(iv) How heavily the generator is loaded.
(v) The generator output during the fault. This depends upon the fault location and type.
(vi) The fault clearing time.
(vii) The postfault transmission system reactance.

5.11 Techniques for Transient Stability Improvement

(i) Reduction in system transfer reactance

Reducing the series reactance by using the series capacitor is normally economical for lines of length more than 320 km. For lines of length less than 320 km, the objective is achieved by running parallel lines. When parallel lines are used, instead of a single line, some power can be transferred over the healthy line even during a three-phase fault on one of the lines, unless of course when a fault takes place at the paralleling bus when no power can be transferred out of the parallel lines. For other types of faults on more than one line, more power can be transferred during the fault, if there are two lines in parallel, power can be transferred over a single faulted line.

The effect of reducing the series reactance is to increase P_m, which therefore, increases the transient stability of a system.

(ii) Increase of system voltage

From the equation $P_{\max} = E_G E_M / X_T$, it is clear that an increase in system voltage results in higher values of power $P_{\max}$ that can be transferred between the nodes. Since shaft power $P_e = P_{\max} \sin \delta_0$, with higher values of $P_{\max}$, δ_0 is reduced and, therefore, the difference between the critical clearing angle and the initial angle δ_0 is increased. Therefore, increasing $P_{\max}$ allows the machine to rotate through large angle before it reaches the critical clearing angle, which results in greater critical clearing time and the probability of maintaining stability.

(iii) Use of high speed reclosing breakers

The quicker a breaker operates, the faster the fault is removed from the system and the better is the tendency of the system to restore to normal operating conditions. The use of high-speed breakers has materially improved the transient stability of the power systems and does not require any other methods for the purpose. The use of reclosing type circuit breakers plays a vital role in improving the transient stability limit.

(iv) HVDC links

A dc link is asynchronous, i.e. the two ac systems at either end do not have to be controlled in phase or even be at exactly the same frequency as they do for an ac link. There is no risk of a fault in one system causing loss of stability in the other system.

(v) Braking resistors

For improving stability, when large load is suddenly lost, a resistive load called a braking resistor is connected at or near the generator bus. This load compensates for at least some of the reduction of load on the generators and so reduces the acceleration. During a fault, the resistors are applied to the terminals of the generators through circuit breakers by means of elaborate control schemes. The control scheme determines the amount of resistance to be applied and its duration.

(vi) Short circuit current limiters

These may be used in long transmission lines to modify favourably the transfer impedance during the fault conditions so that the voltage profile of the system is somewhat improved, thereby raising the system load level during the fault.

(vii) Turbine fast valving or bypass valving

Another recent method of improving the stability of a unit is to decrease the mechanical input power to the turbine. This can be accomplished by means of fast valving, where the difference between the mechanical input and the reduced electrical output of a generator under a fault, as sensed by a control scheme, initiates the closing of a turbine valve to reduce the power input.

(viii) Full load rejection technique

In places where stability is difficult to maintain, the normal procedure is to automatically trip the unit off the line. This, however, causes several hours of delay before the unit can be put back into operation. The loss of a major unit for this length of time can seriously jeopardize the remaining system. To remedy these situations, a full load rejection scheme could be utilized after the unit is separated from the system. To do this, the unit has to be equipped with a large steam bypass system. After the system has recovered from the shock caused by the fault, the unit could be resynchronized and reloaded. The main disadvantage of this method is the extra cost of a large bypass system.

Review Questions

Part-A

1. What is the power system stability?
2. How is the power system stability classified?
3. What is the rotor angle stability?
4. What is the steady state stability?
5. What is the steady state stability limit?
6. What is the transient stability?
7. What is the transient stability limit?
8. What is the dynamic stability?
9. What is the voltage stability?
10. State the causes of voltage instability.
11. Write the power angle equation and draw the power angle curve.
12. Write the expression for the maximum power transfer.
13. Write the swing equation for a SMIB (single machine connected to an infinite bus bar) system.

14. Define the swing curve.

15. In a three machine system having ratings G_1, G_2 and G_3 and inertia constants M_1, M_2 and M_3 what are the inertia constants M and H of the equivalent system.

16. State the assumptions made in stability studies.

17. State equal area criterion.

18. Define the critical clearing angle.

19. List the methods of improving the transient stability limit of a power system.

20. What are the numerical integration methods of power system stability?

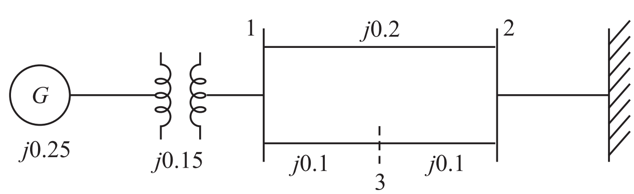
Part-B

1. A 400 MVA synchronous machine has H_1 = 4.6 MJ/MVA and a 1200 MVA machine H_2 = 3.0 MJ/MVA. Two machines operate in parallel in a power plant. Find out H_{eq} relative to a 100 MVA base.

2. A 100 MVA, two pole, 50 Hz generator has moment of inertia 40×10^3 kg·m^2. What is the energy stored in the rotor at the rated speed? What is the corresponding angular momentum? Determine the inertia constant H.

3. The sending end and the receiving end voltages of a three-phase transmission line at a 200 MW load are equal at 230 kV. The per phase line impedance is $j14\ \Omega$. Calculate the maximum steady state power that can be transmitted over the line.

4. A single line diagram of a system is shown in the figure below. All the values are in per unit on a common base. The power delivered into bus 2 is 1.0 p.u. at 0.80 power factor lagging. Obtain the power angle equation and the swing equation for the system. Neglect all losses.



5. A 50 Hz synchronous generator capable of supplying 400 MW of power is connected to a larger power system and is delivering 80 MW when a three-phase fault occurs at its terminals, determine (a) the time in which the fault must be cleared if the maximum power angle is to be –85°, assume H = 7 MJ/MVA on a 100 MVA base and (b) the critical clearing angle.

6. A synchronous generator is connected to a large power system and supplying 0.45 p.u. MW of its maximum power capacity. A three-phase fault occurs and the effective terminal voltage of the generator becomes 25% of its value before the fault. When the fault is cleared, the generator is delivering 70% of the original maximum value. Determine the critical clearing angle.

7. Determine the critical clearing angle of the power system as shown in the figure below for a three-phase fault at the point *F*. The generator is supplying 1.0 p.u. MW power under the prefault condition.